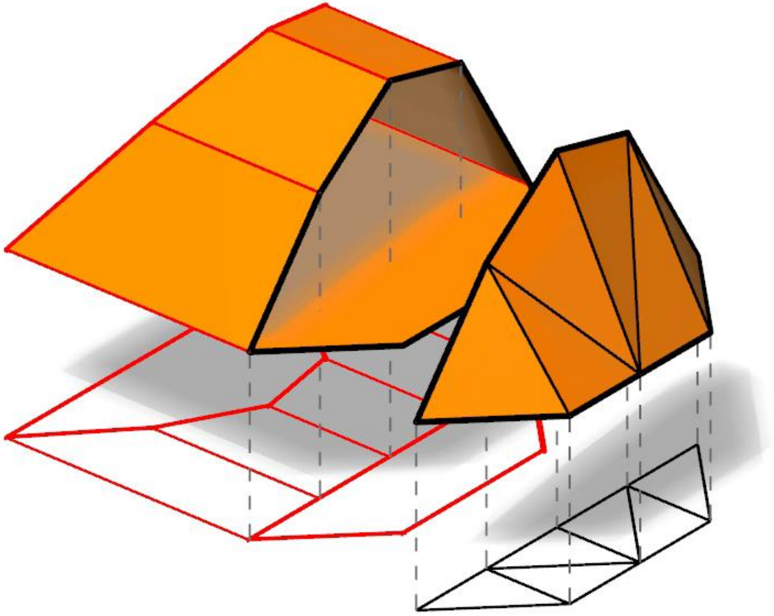
Theory and the Creation of Structures



Stålbyggnadsdagen
Gothenburg
7 November 2024

Bill Baker, NAE, FREng, Dist.M.ASCE, PE, SE, C.Eng
Consulting Partner
Skidmore, Owings & Merrill

The Challenge is
to Create Ideas and Technologies
that will lead to Efficient Structures.

FOCUS ON THE GEOMETRY OF STRUCTURES AND WHY GEOMETRY MATTERS


Structural systems are essential for efficiency

Geometry is critical to structural systems

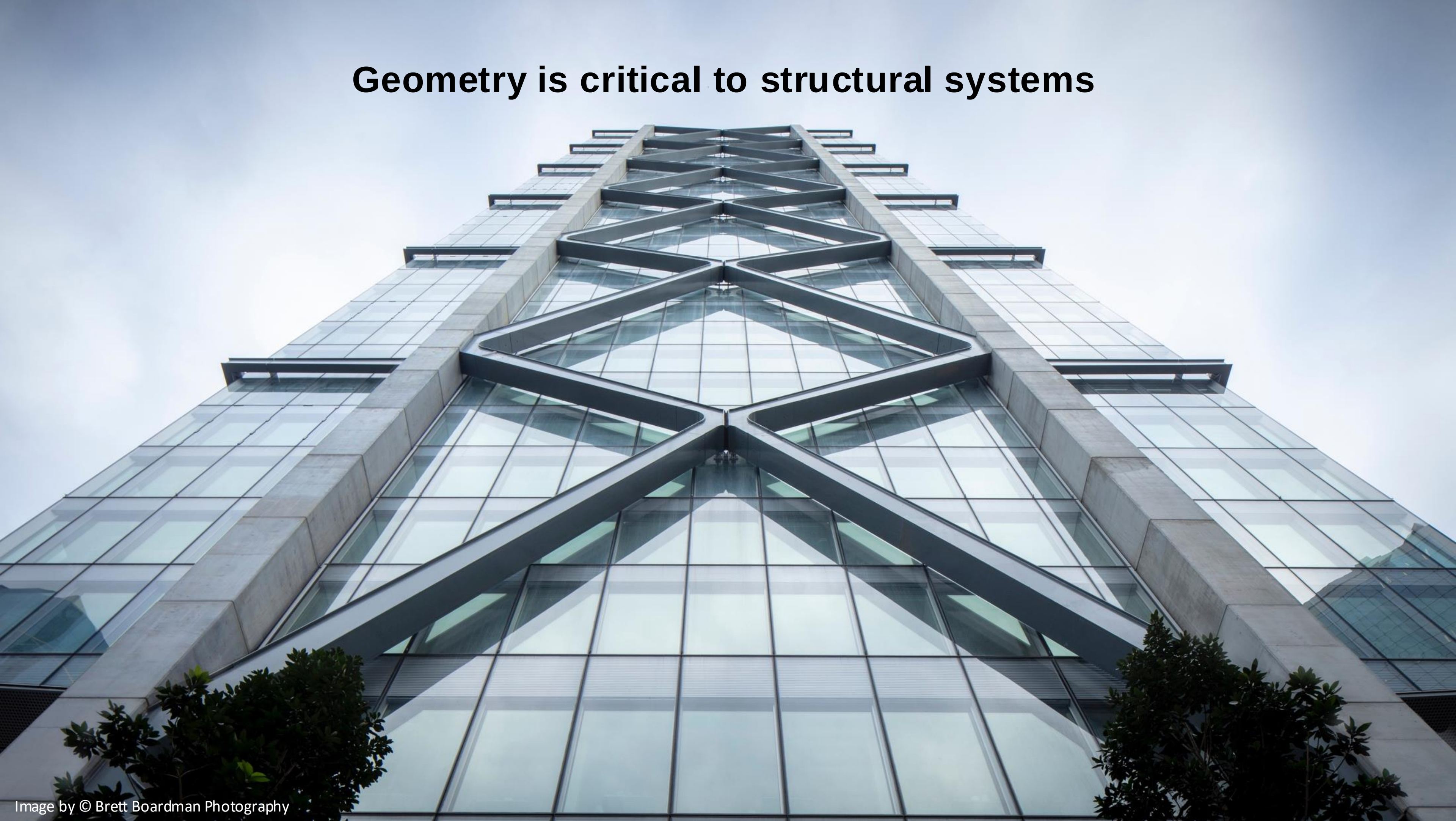
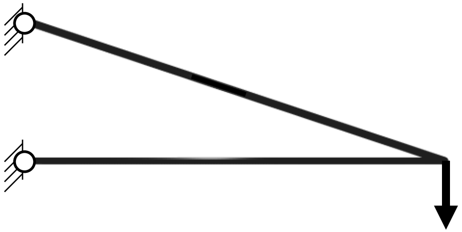
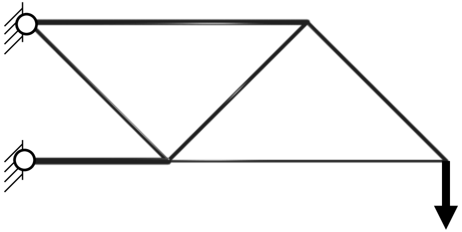


Image by © Brett Boardman Photography

Geometry is the Key to Sustainability



Truss "A"

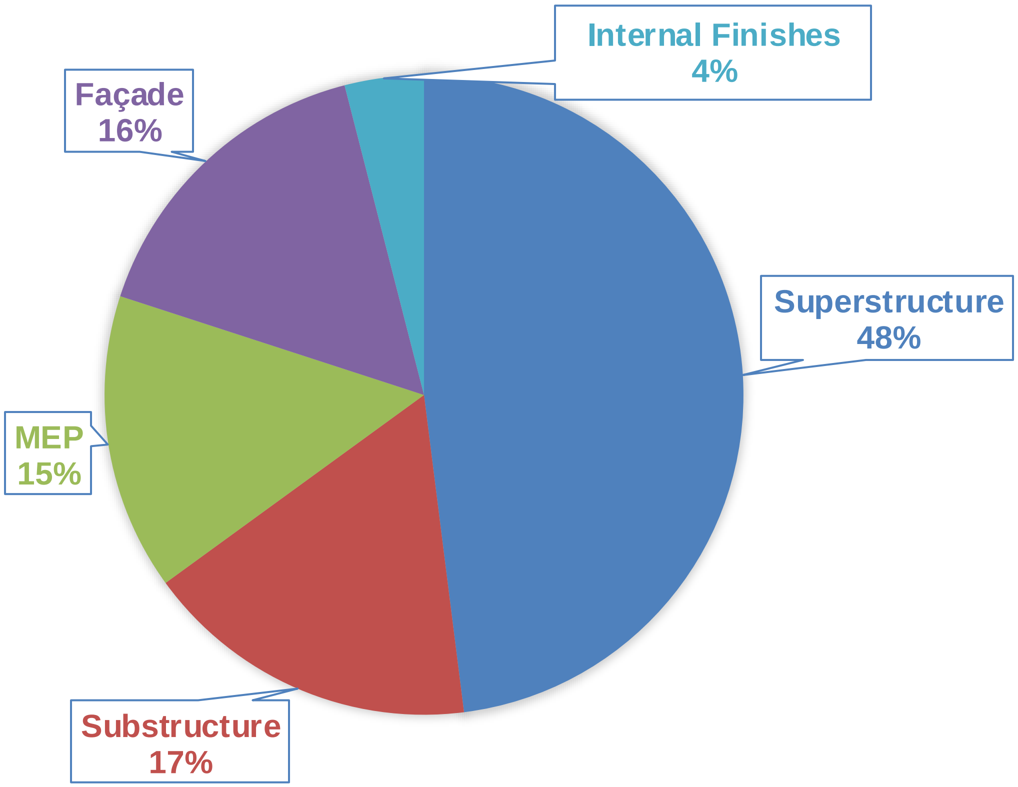


Truss "B"

Strength $\frac{V_A}{V_B} =$ 27% More

Deflection $\frac{V_A}{V_B} =$ 60% More

Efficient structures consume less resources



Carbon on Day One

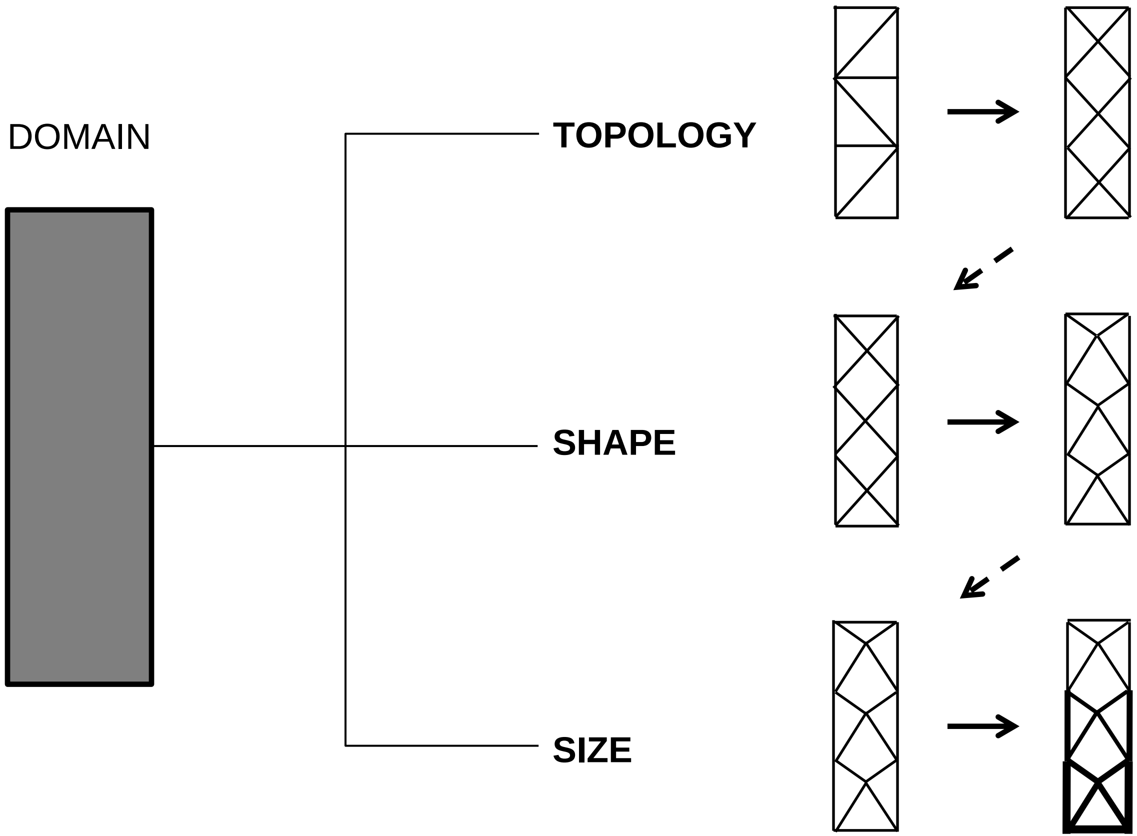
Source: *London Energy Transformation Initiative (LETI) Embodied Carbon Primer*

GEOMETRY IS THE INTERSECTION
OF ARCHITECTURE AND STRUCTURE

TERMINOLOGY

TOPOLOGY – SHAPE - SIZE

Geometry Terminology for Design



HOW CAN ONE CREATE EFFICIENT STRUCTURAL SYSTEMS?

BUILD ON EXISTING IDEAS

Notable Structural Designers-Engineers, Architects & Artists

Othmar Ammann
Ove Arup
Benjamin Baker
Áine Brazil
Isambard Kingdom Brunel
Santiago Calatrava
Eduardo Catalano
Felix Candela
Jamie Carpenter
Jörg Conzett
Mike Cook
Abraham Darby III
Eladio Dieste
James Eads
Janet Echelman
Gustave Eiffel
Aldo Favini
Ulrich Finsterwalder
Miguel Fisac
Eugene Freyssinet
Buckminster Fuller

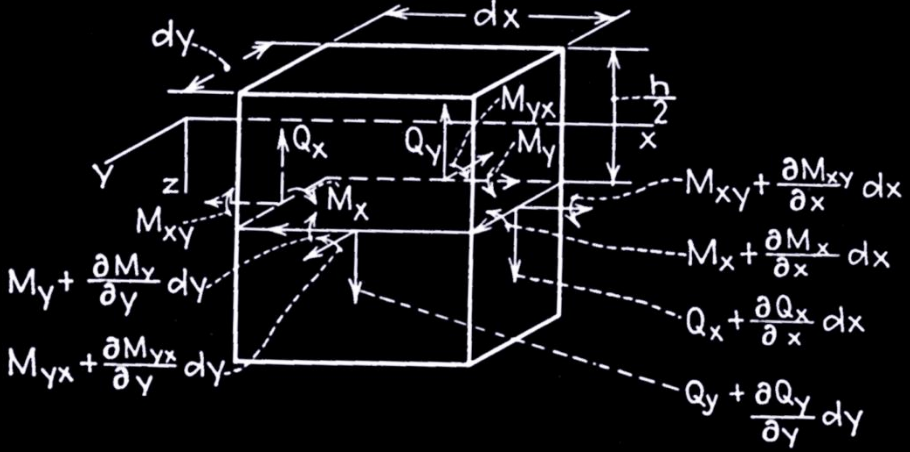
Antonio Gaudi
Myron Goldsmith
Rafael Guastavino
Edmund Happold
Chuck Hoberman
Tony Hunt
Heinz Isler
Theo Jansen
Mamoru Kawaguchi
Gertrude Kerbis
Fazlur Khan
Martin Knight
Jan Knippers
Ian Liddell
William LeMessurier
Fritz Leonhardt
Robert LeRicolais
Robert Maillart
Christian Menn
A.G.M. Michell
Marc Mimram

Leon Moissieff
Jean Muller
Sergio Musmeci
Ulrich Müther
Pier Luigi Nervi
Laurent Ney
Isamu Noguchi
Guy Nordenson
Frei Otto
Kate Purver
Mahendra Raj
Julia Ratcliffe
Ron Resch
Peter Rice
John Roebling
Emily Warren Roebling
Les Robertson
José Romo Martín
Mutsuro Sasaki
Jörg Schlaich
Mike Schlaich

Hans Schober
Kenneth Snelson
Werner Sobek
Juan Sobrino
Juri Strasky
Vladimir Suchov
Yoshikatsu Tsuboi
Anton Tedesko
Thomas Telford
Eduardo Torroja
Michel Virlogeux
Konrad Wachsmann
Jane Wernick
Chris Williams
Chris Wise
Waclaw Zalewski

LOOK TO THEORY FOR DESIGN IDEAS

Theory and Behavior



A FEW GOOD THEORIES

MAXWELL LOAD PATH THEOREM
1870

James Clerk Maxwell
1831 - 1879

[From the *Transactions of the Royal Society of Edinburgh*, Vol. XXVI.]

XXXIX. *On Reciprocal Figures, Frames, and Diagrams of Forces.*

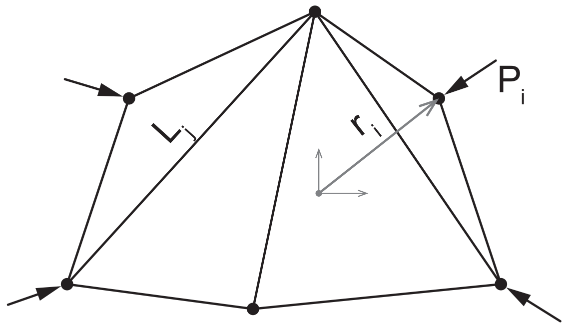
(Received 17th Dec. 1869; read 7th Feb. 1870.)

Two figures are reciprocal when the properties of the first relative to the second are the same as those of the second relative to the first. Several kinds of reciprocity are known to mathematicians, and the theories of Inverse Figures and of Polar Reciprocals have been developed at great length, and have led to remarkable results. I propose to investigate a different kind of geometrical reciprocity, which is also capable of considerable development, and can be applied to the solution of mechanical problems.

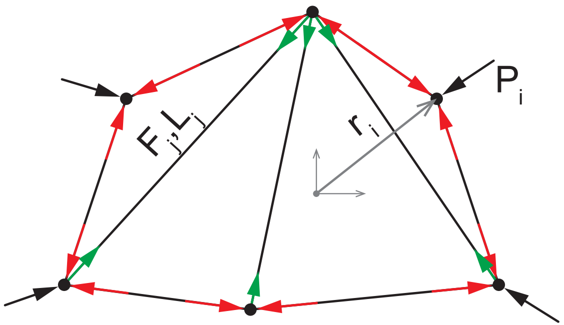
A Frame may be defined geometrically as a system of straight lines connecting a number of points. In actual structures these lines are material pieces, beams, rods, or wires, and may be straight or curved; but the force by which each piece resists any alteration of the distance between the points which it joins acts in the straight line joining those points. Hence, in studying the equilibrium of a frame, we may consider its different points as mutually acting on each other with forces whose directions are those of the lines joining each pair of points. When the forces acting between the two points tend to draw them together, or to prevent them from separating, the action along the joining line is called a Tension. When the forces tend to separate the points, or to keep

MAXWELL PROOF
EXTERNALLY LOADED TRUSS

Maxwell Load Path Theorem – external loads

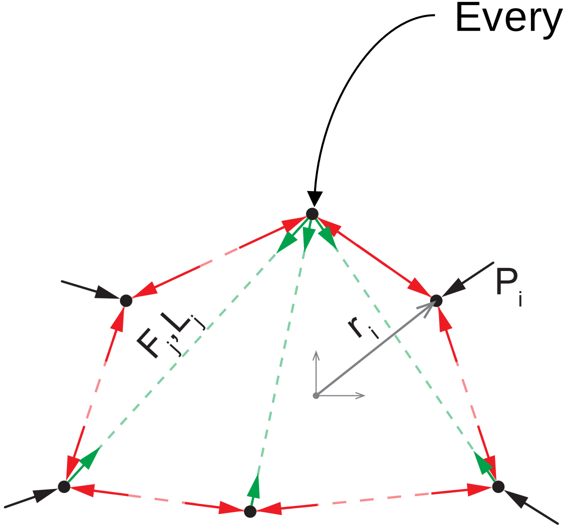


Maxwell Load Path Theorem – external loads



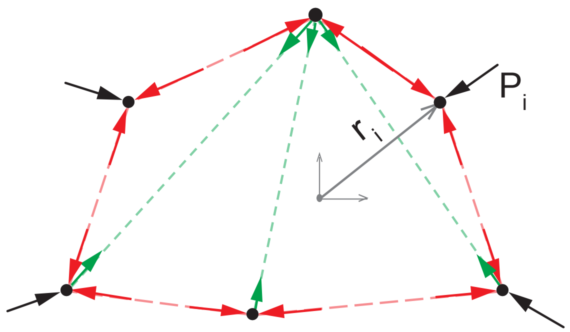
Maxwell Load Path Theorem – external loads

Every node is in equilibrium



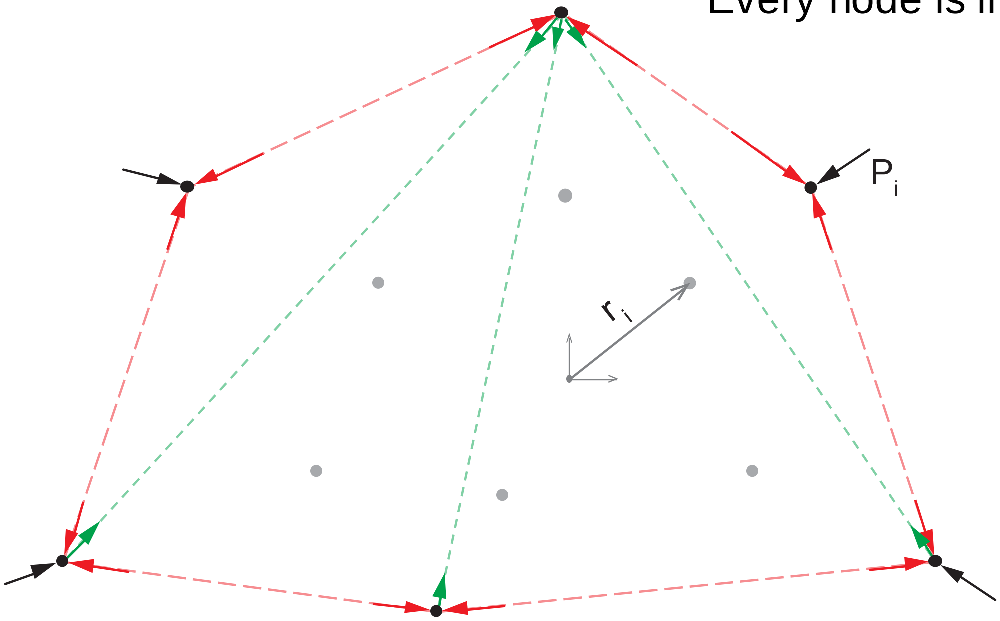
Maxwell Load Path Theorem – external loads

Every node is in equilibrium



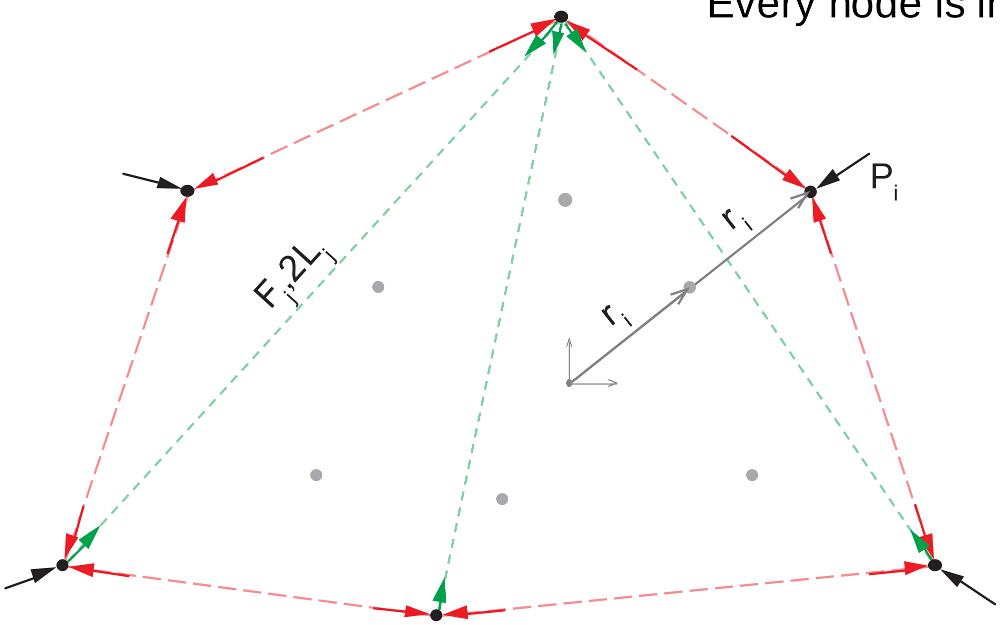
Maxwell Load Path Theorem – external loads

Every node is in equilibrium



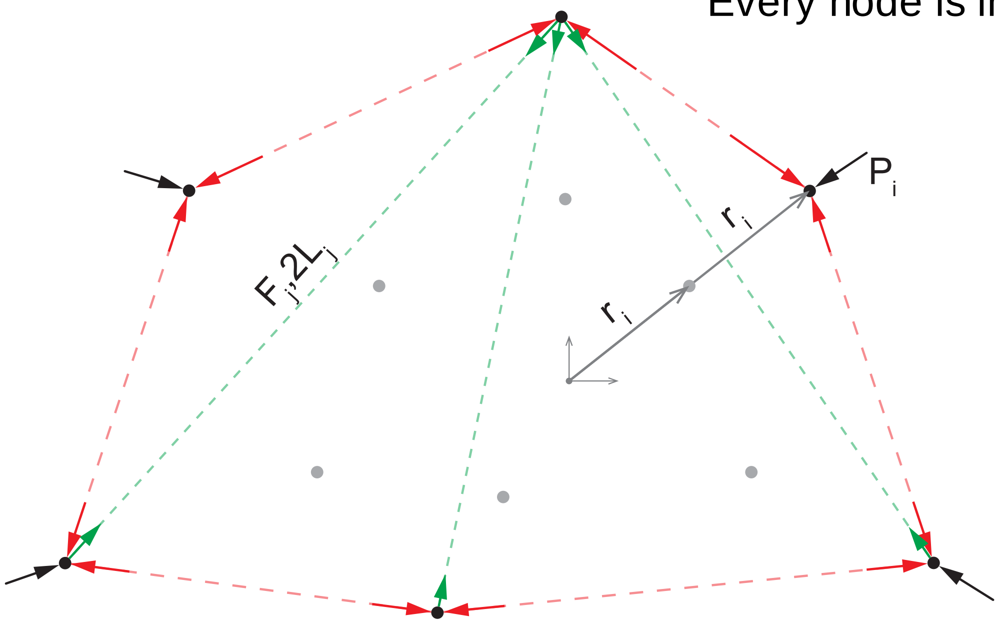
Maxwell Load Path Theorem – external loads

Every node is in equilibrium



Maxwell Load Path Theorem – external loads

Every node is in equilibrium

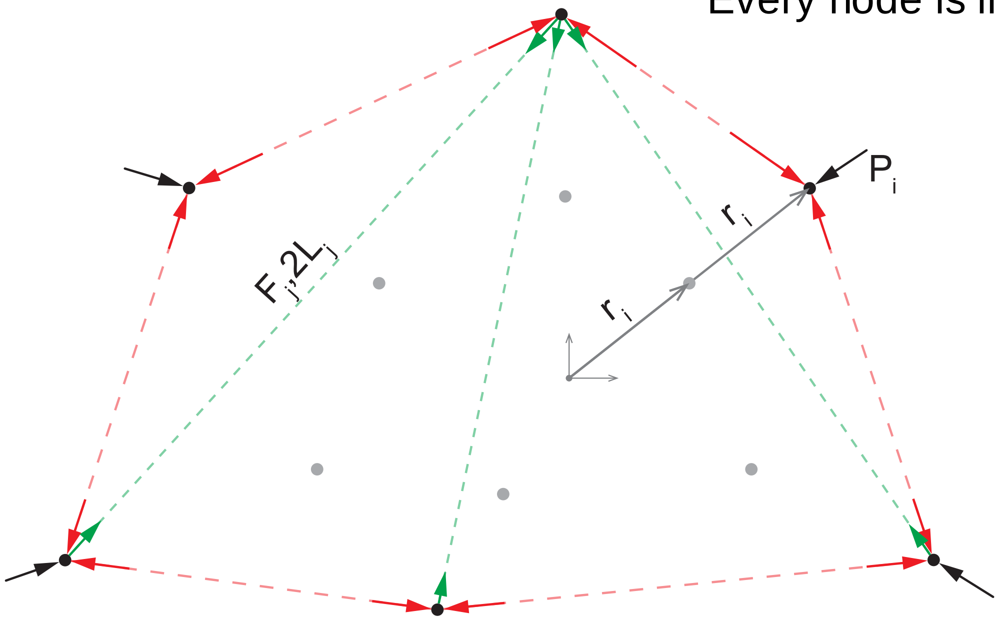


$$External\ Work = \sum \vec{P} \cdot \vec{r}$$

$$Internal\ Work = \sum F_C L_C - \sum F_T L_T$$

Maxwell Load Path Theorem – external loads

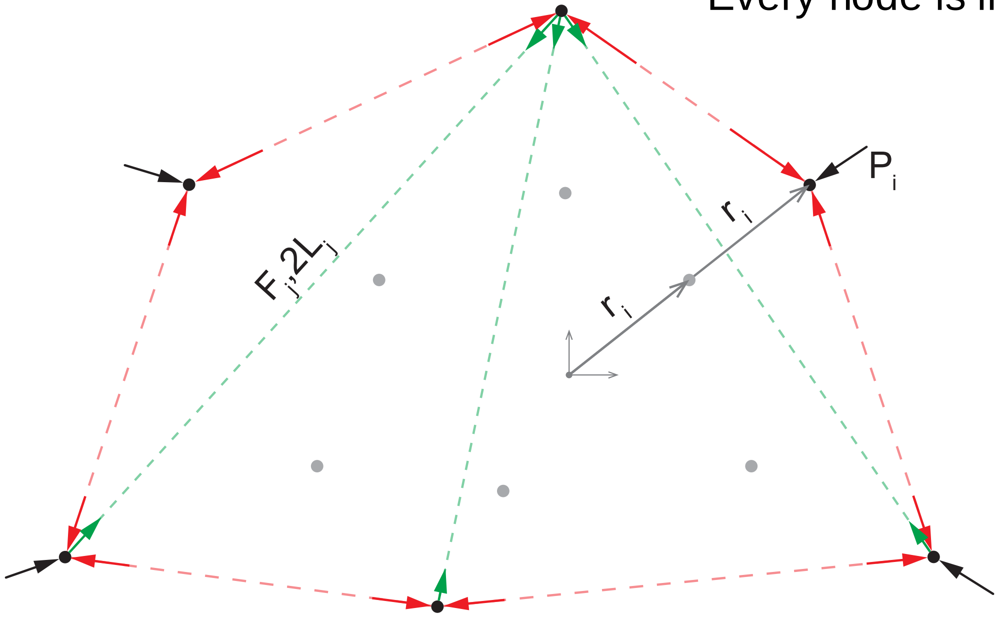
Every node is in equilibrium



$$\sum F_C L_C - \sum F_T L_T + \sum \vec{P} \cdot \vec{r} = 0$$

Maxwell Load Path Theorem – external loads

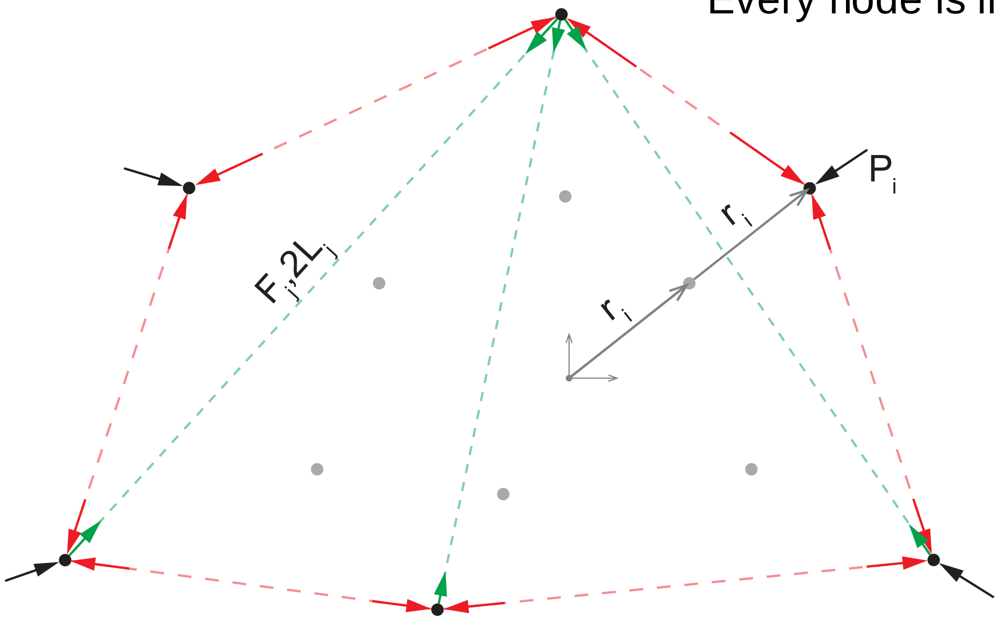
Every node is in equilibrium



$$\sum F_T L_T - \sum F_C L_C = \sum \vec{P} \cdot \vec{r}$$

Maxwell Load Path Theorem – external loads

Every node is in equilibrium

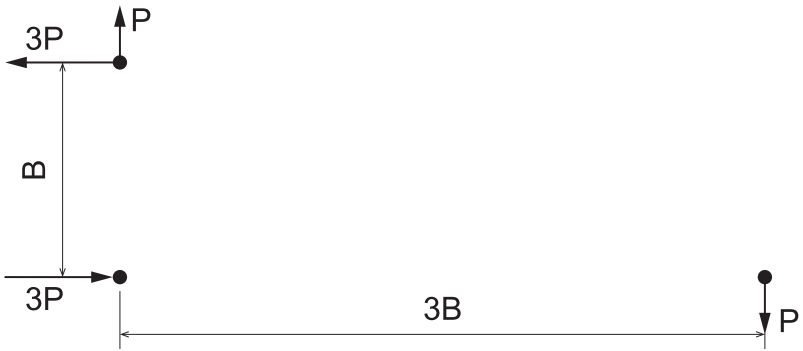


$$\sum F_T L_T - \sum F_C L_C = \sum \vec{P} \cdot \vec{r} = \text{ Maxwell Number}$$

MAXWELL LOAD PATH THEOREM
CANTILEVER EXAMPLE

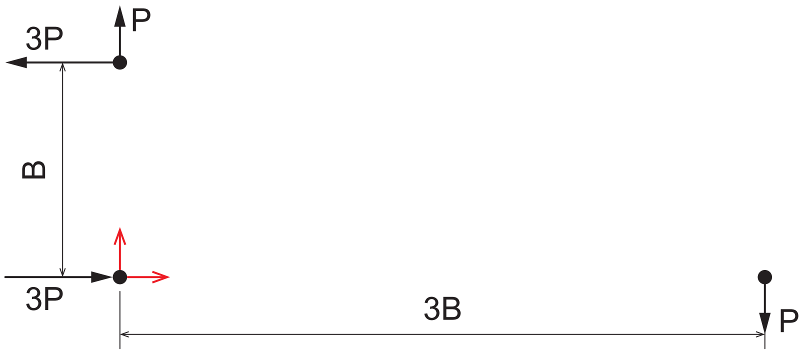
Maxwell Load Path Theorem – example

Cantilever with 3 to 1 span



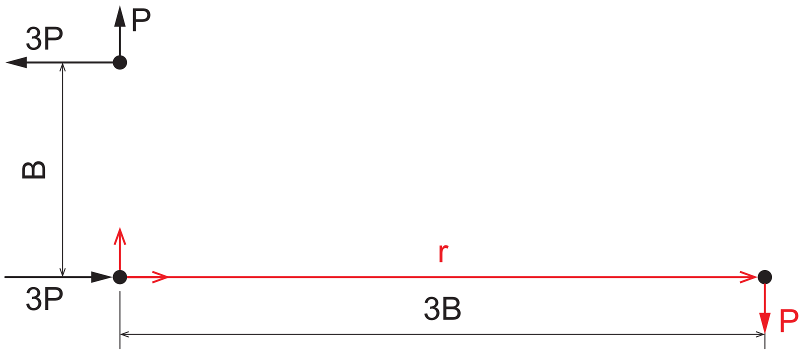
Maxwell Load Path Theorem – example

Cantilever with 3 to 1 span



Maxwell Load Path Theorem – example

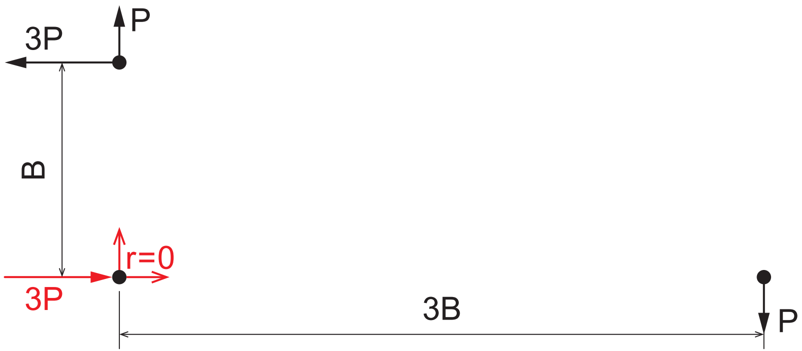
Cantilever with 3 to 1 span



$$\vec{P} \cdot \vec{r} = 0$$

Maxwell Load Path Theorem – example

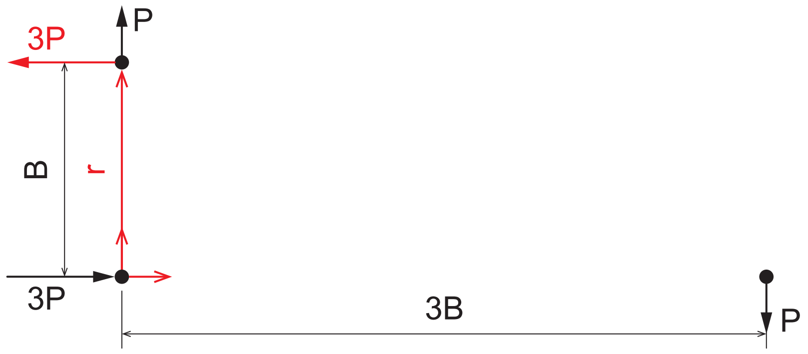
Cantilever with 3 to 1 span



$$\vec{P} \cdot \vec{r} = 0$$

Maxwell Load Path Theorem – example

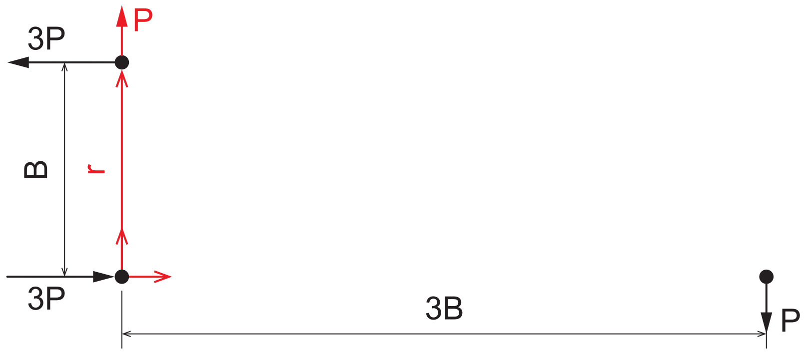
Cantilever with 3 to 1 span



$$\vec{P} \cdot \vec{r} = 0$$

Maxwell Load Path Theorem – example

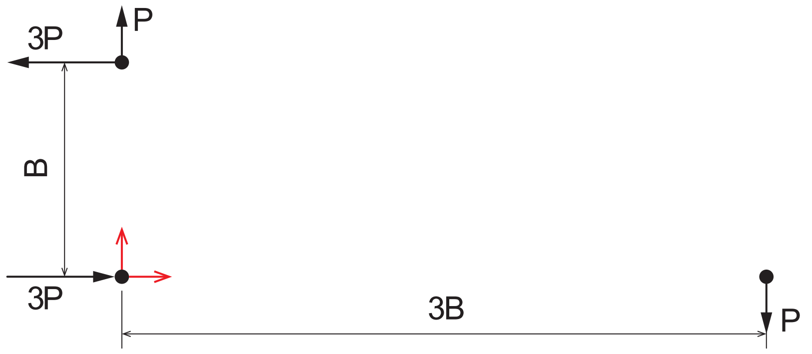
Cantilever with 3 to 1 span



$$\vec{P} \cdot \vec{r} = PB$$

Maxwell Load Path Theorem – example

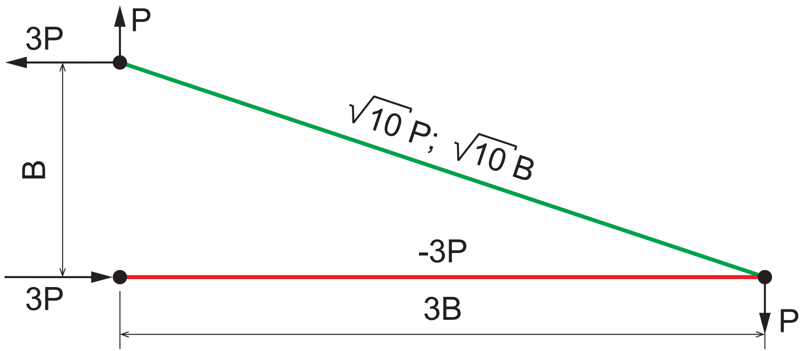
Cantilever with 3 to 1 span



$$\sum \vec{P} \cdot \vec{r} = PB$$

Maxwell Load Path Theorem – example

Moment diagram truss geometry

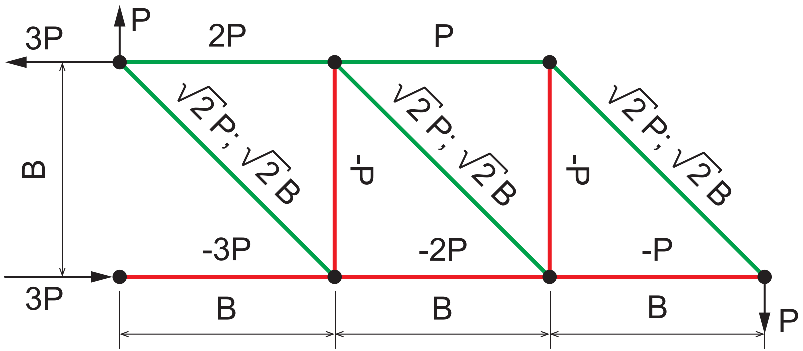


$$\sum F_T L_T = 10PB \qquad \sum F_T L_T - \sum F_C L_C = PB \qquad \Delta = \boxed{19}\frac{\sigma B}{E}$$

$$\sum F_C L_C = 9PB \qquad \sum F_T L_T + \sum F_C L_C = \boxed{19}PB \qquad V = \boxed{19}\frac{PB}{\sigma}$$

Maxwell Load Path Theorem – example

Pratt truss:

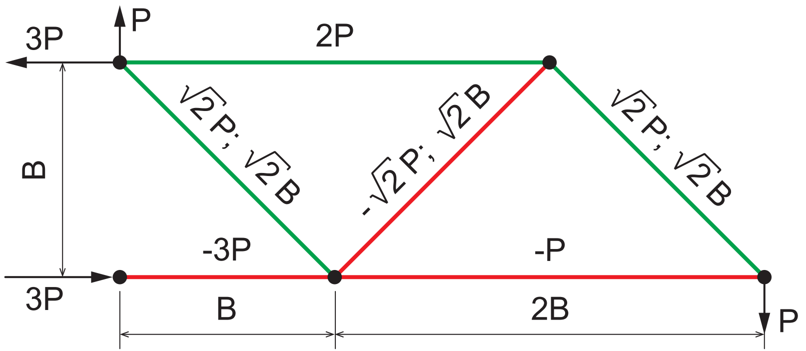


$$\sum F_T L_T = 9PB \qquad \sum F_T L_T - \sum F_C L_C = PB \qquad \Delta = \mathbf{17}\frac{\sigma B}{E}$$

$$\sum F_C L_C = 8PB \qquad \sum F_T L_T + \sum F_C L_C = \mathbf{17}PB \qquad V = \mathbf{17}\frac{PB}{\sigma}$$

Maxwell Load Path Theorem – example

Warren truss:



$$\sum F_T L_T = 8PB$$

$$\sum F_T L_T - \sum F_C L_C = PB$$

$$\Delta = 15\frac{\sigma B}{E}$$

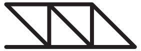
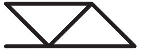
$$\sum F_C L_C = 7PB$$

$$\sum F_T L_T + \sum F_C L_C = 15PB$$

$$V = 15\frac{PB}{\sigma}$$

Maxwell Load Path Theorem – example

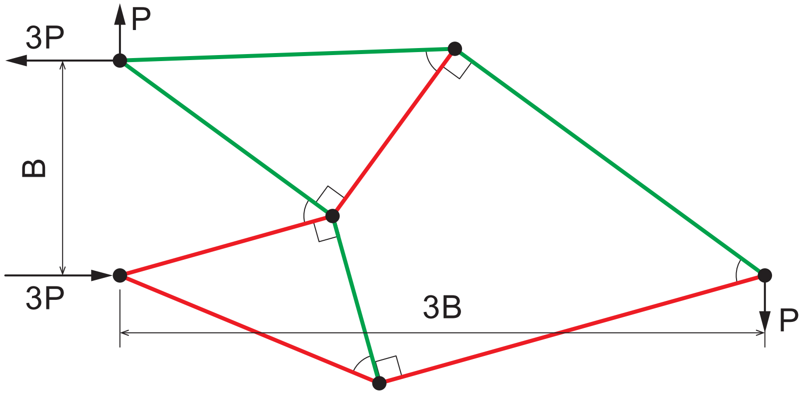
The most material **efficient** truss is also the **stiffest** one!

Truss	Tensile Load Path $\sum F_T L_T$	Compressive Load Path $\sum F_C L_C$	Difference in Load Paths $\sum F_T L_T - \sum F_C L_C$	Sum of Load Paths $\sum F_T L_T + \sum F_C L_C$	Deflection Δ
Moment diagram 	10PB	9PB	PB	19PB	19 σB/E
Pratt 	9PB	8PB	PB	17PB	17 σB/E
Warren 	8PB	7PB	PB	**15PB**	**15 σB/E**

Can we find a benchmark for our design?

Maxwell Load Path Theorem – Discrete Michell Truss Example 1

How low can we go?

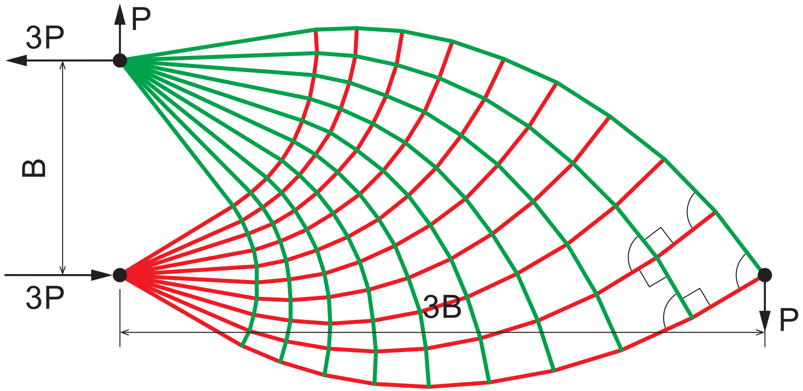


$$\sum F_T L_T = 7.46PB \qquad \sum F_T L_T - \sum F_C L_C = PB \qquad \Delta = 13.92 \frac{\sigma B}{E}$$

$$\sum F_C L_C = 6.46PB \qquad \sum F_T L_T + \sum F_C L_C = 13.92PB \qquad V = 13.92 \frac{PB}{\sigma}$$

Maxwell Load Path Theorem – Discrete Michell Truss Example 2

How low can we go?



$$\sum F_T L_T = 7.08PB$$

$$\sum F_C L_C = 6.08PB$$

$$\sum F_T L_T - \sum F_C L_C = PB$$

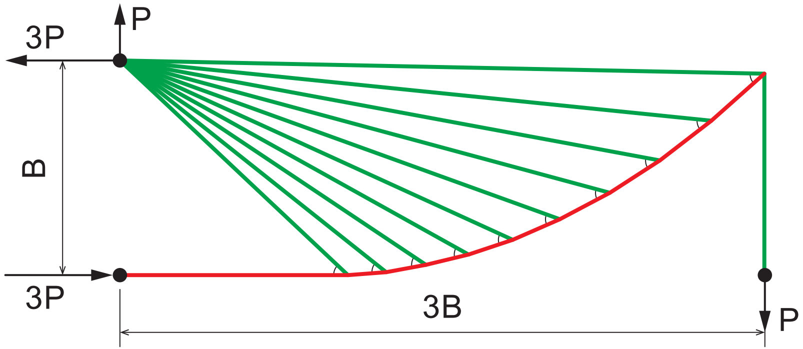
$$\sum F_T L_T + \sum F_C L_C = \mathbf{13.17}PB$$

$$\Delta = \mathbf{13.17}\frac{\sigma B}{E}$$

$$V = \mathbf{13.17}\frac{PB}{\sigma}$$

Maxwell Load Path Theorem – Discrete Michell Truss Example 3

Other solutions

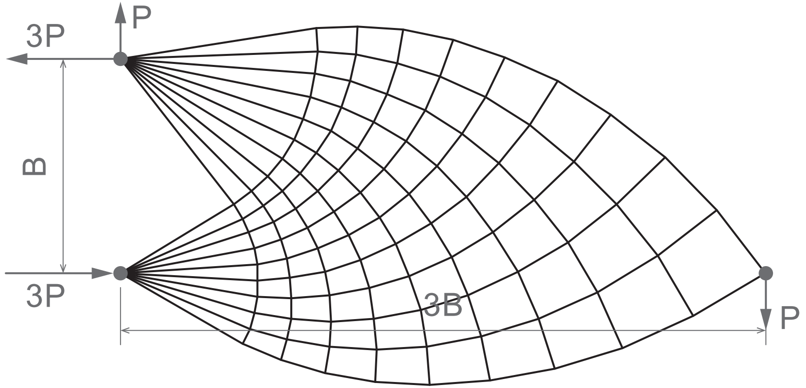


$$\sum F_T L_T = 8.52PB \qquad \sum F_T L_T - \sum F_C L_C = PB \qquad \Delta = 16.04\frac{\sigma B}{E}$$

$$\sum F_C L_C = 7.52PB \qquad \sum F_T L_T + \sum F_C L_C = 16.04PB \qquad V = 16.04\frac{PB}{\sigma}$$

Maxwell Load Path Theorem – example

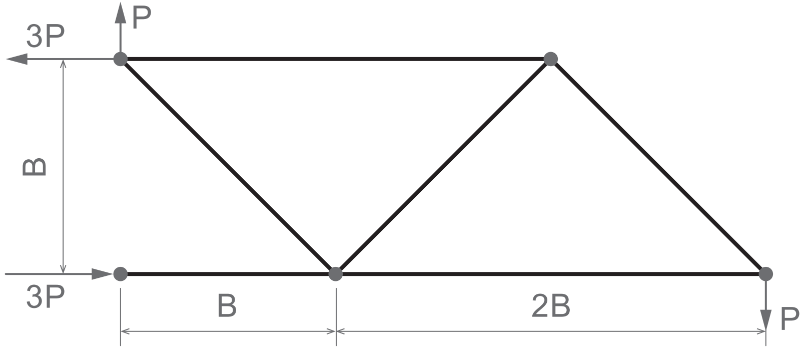
Most optimum truss:



Benchmark Load Path

$$\sum F_T L_T + \sum F_C L_C = \mathbf{13.17} PB$$

Warren truss:

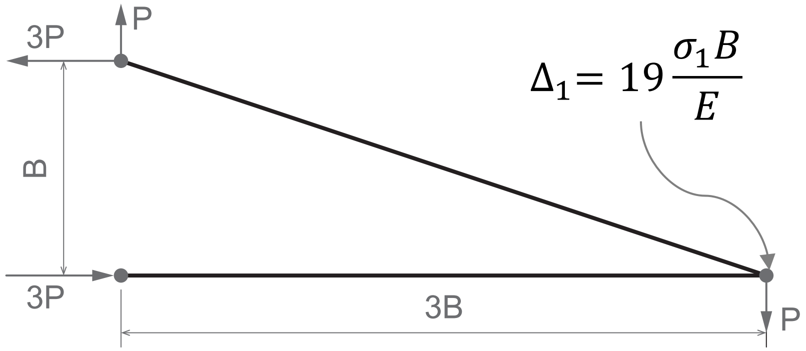


Within 14% of the benchmark

$$\sum F_T L_T + \sum F_C L_C = \mathbf{15} PB$$

Maxwell Load Path Theorem – Equal deflection

moment diagram truss geometry



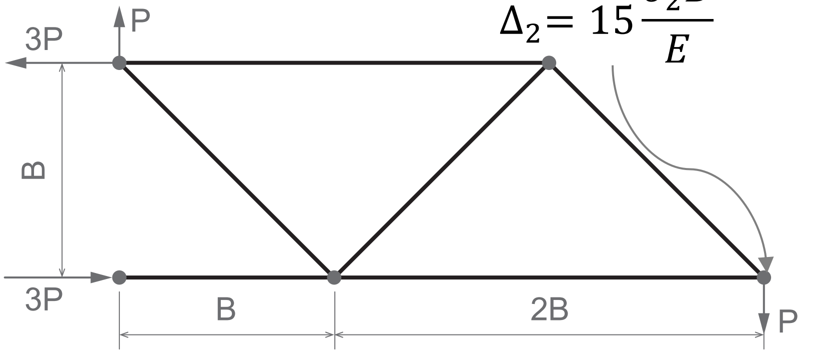
$$\Delta_1 = 19\frac{\sigma_1 B}{E}$$

$$V_1 = 19\frac{PB}{\sigma_1}; \sigma_1 = \frac{\Delta_1 E}{19B}$$

$$V_1 = 19\frac{PB}{\frac{\Delta_1 E}{19B}} = 19^2\frac{PB^2}{\Delta_1 E}$$

warren truss geometry

$$\Delta_2 = 15\frac{\sigma_2 B}{E}$$



$$V_2 = 15\frac{PB}{\sigma_2}; \sigma_2 = \frac{\Delta_2 E}{15B}$$

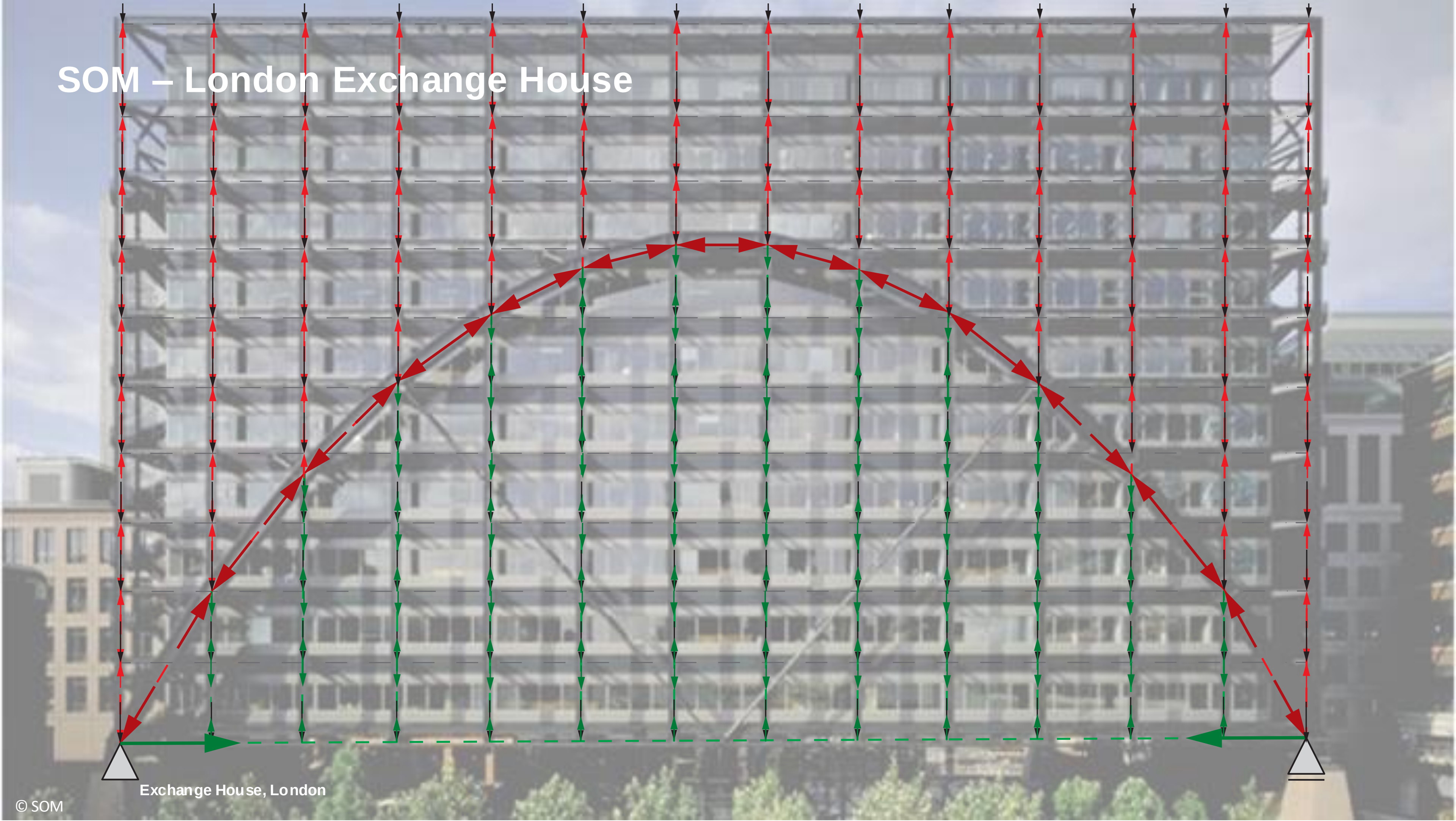
$$V_2 = 15\frac{PB}{\frac{\Delta_2 E}{15B}} = 15^2\frac{PB^2}{\Delta_2 E}$$

$$if \quad \Delta_1 = \Delta_2 \quad \Rightarrow \quad \frac{V_1}{V_2} = \left(\frac{19}{15}\right)^2 = \mathbf{160\%}$$

SOM – London Exchange House

Image by © Richard Waite Photography

SOM – London Exchange House



Exchange House, London

GRAPHIC STATICS

1864

James Clerk Maxwell
1831 - 1879

Maxwell – April 1864

XLV. *On Reciprocal Figures and Diagrams of Forces.* By J. Clerk Maxwell, *F.R.S., Professor of Natural Philosophy in King's College, London* ‡.

RECIPROCAL figures are such that the properties of the first relative to the second are the same as those of the second relative to the first. Thus inverse figures and polar reciprocals are instances of two different kinds of reciprocity.

The kind of reciprocity which we have here to do with has reference to figures consisting of straight lines joining a system of points, and forming closed rectilinear figures; and it consists in the directions of all lines in the one figure having a constant relation to those of the lines in the other figure which correspond to them.

In plane figures, corresponding lines may be either parallel,

Maxwell – April 1864

XLV. *On Reciprocal Figures and Diagrams of Forces.* By J. Clerk Maxwell, *F.R.S., Professor of Natural Philosophy in King's College, London*‡.

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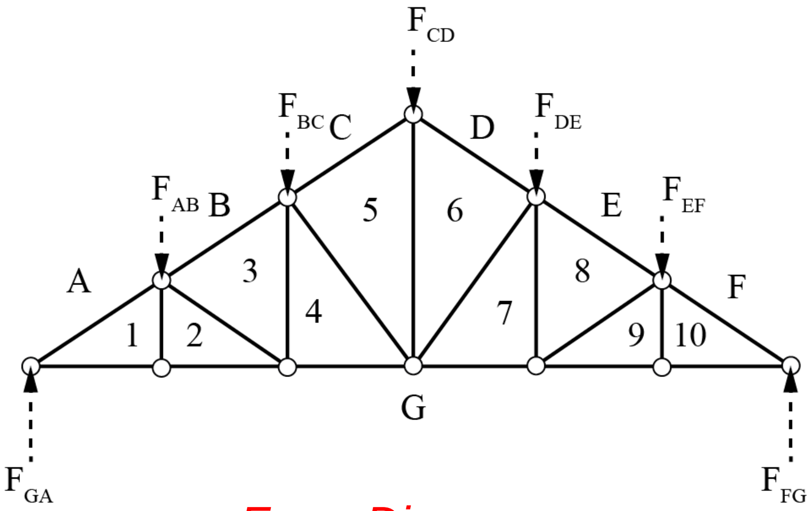
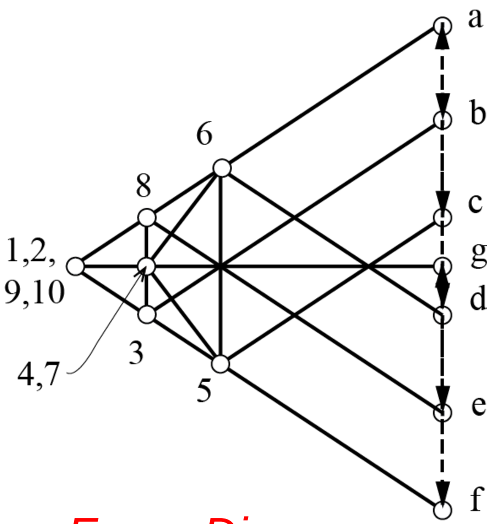
In plane figures, corresponding lines may be either parallel,

GRAPHIC STATICS
AS A DESIGN TOOL

Graphic Statics



Form Diagram

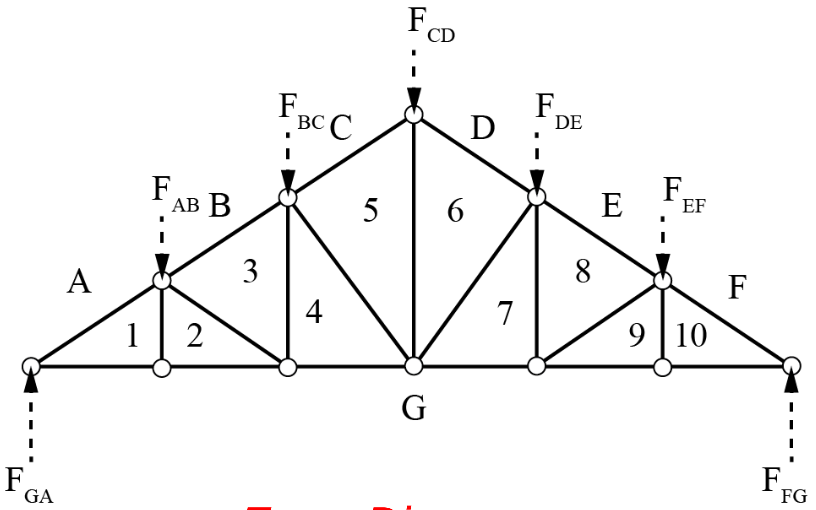


Force Diagram

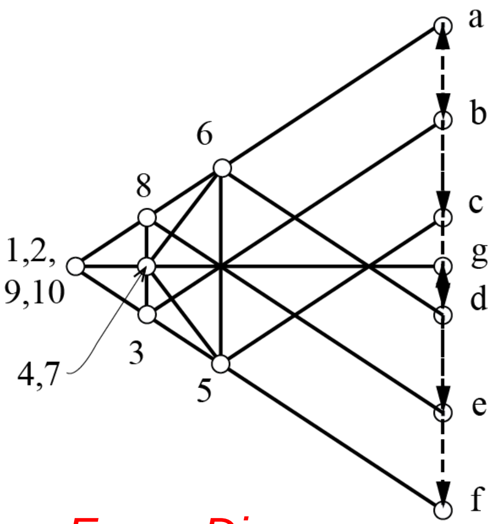
DESIGN THE FORCES WITH GRAPHIC STATICS

Graphic Statics

How can we make the force in the top chord constant?



Form Diagram

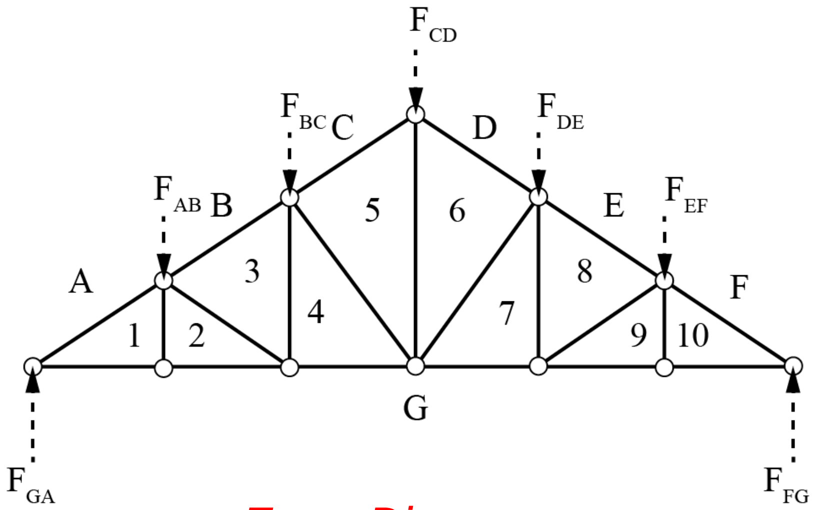


Force Diagram

Modify the force diagram and work backwards!

Graphic Statics

How can we make the force in the top chord constant?



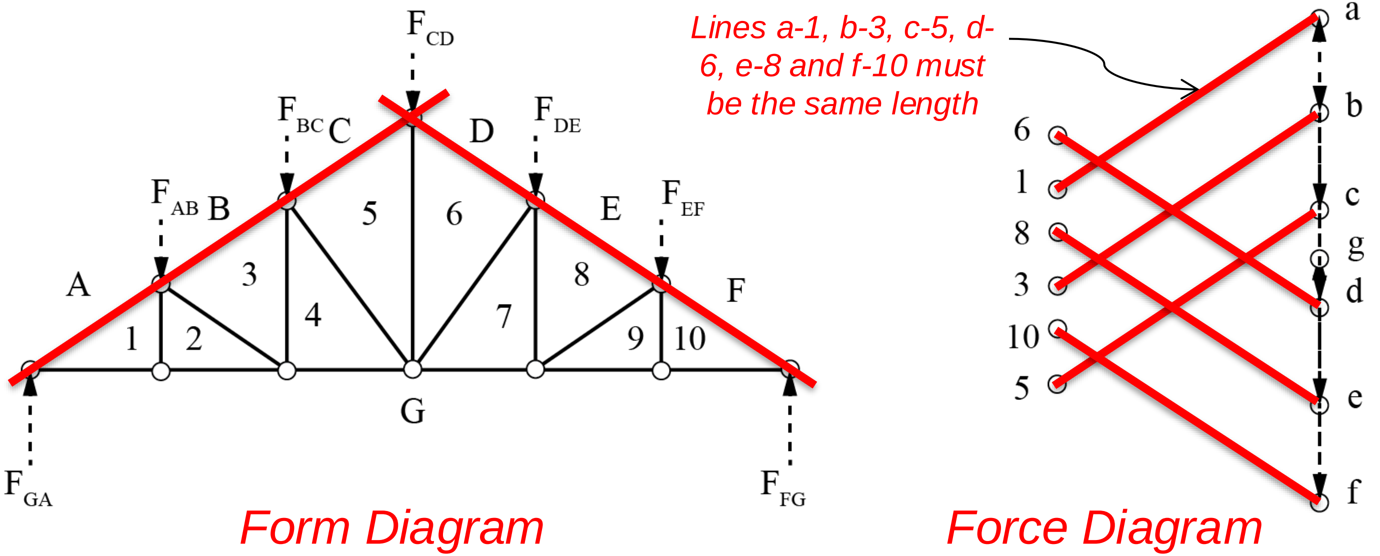
Form Diagram

Force Diagram

Modify the force diagram and work backwards!

Graphic Statics

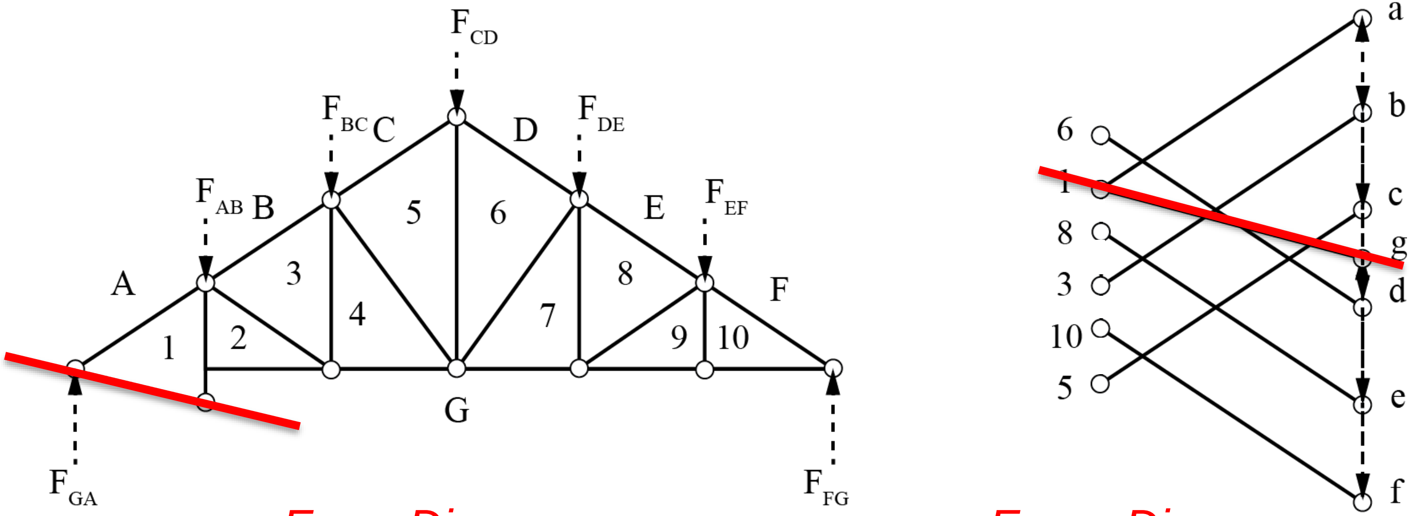
How can we make the force in the top chord constant?



Modify the force diagram and work backwards!

Graphic Statics

How can we make the force in the top chord constant?

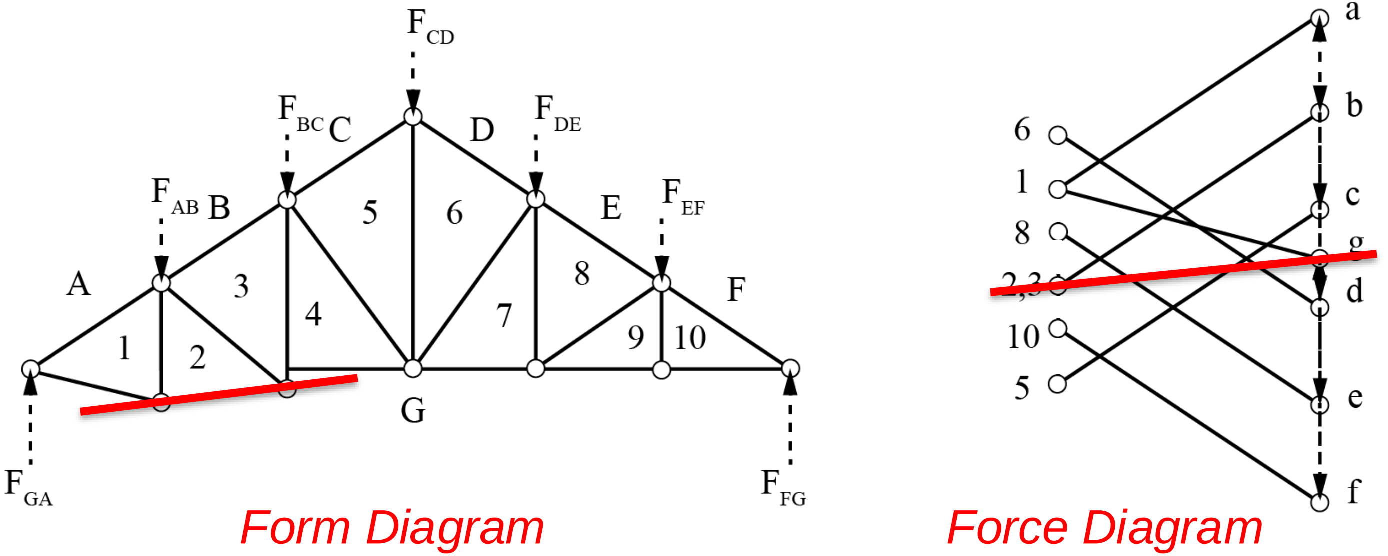


Form Diagram *Force Diagram*

Directions of bottom chord members are defined by connecting point in the force diagram.

Graphic Statics

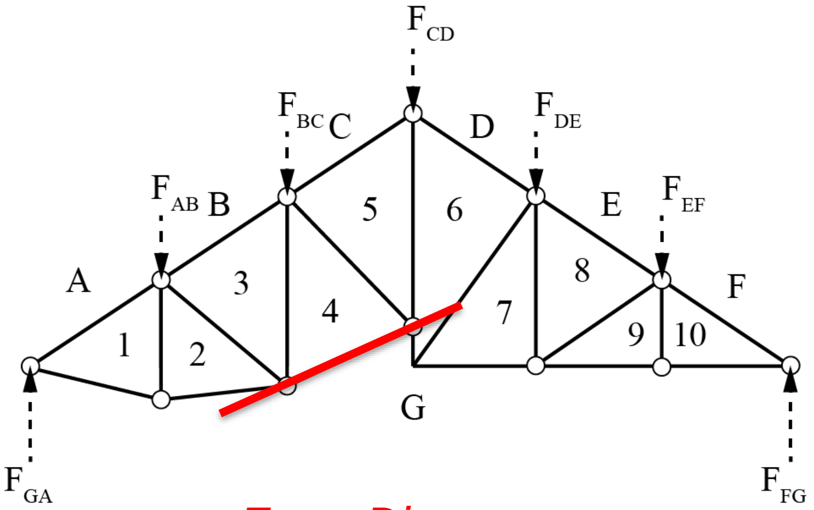
How can we make the force in the top chord constant?



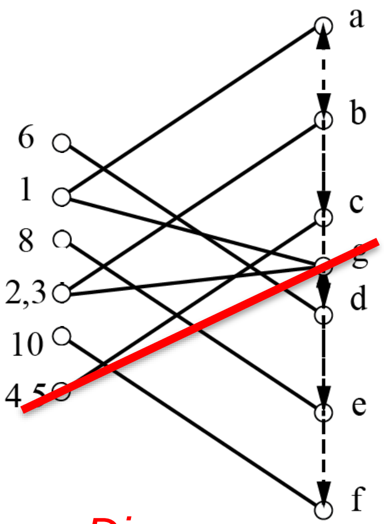
Directions of bottom chord members are defined by connecting point in the force diagram. Diagonal web member 2-3 is made a zero force member.

Graphic Statics

How can we make the force in the top chord constant?



Form Diagram

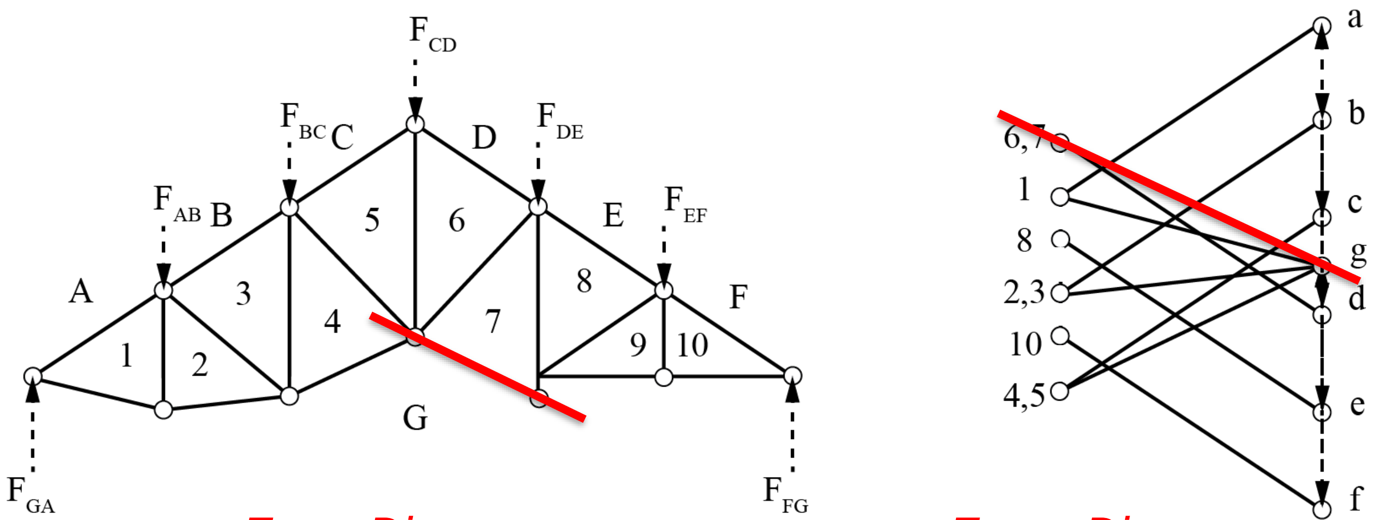


Force Diagram

Directions of bottom chord members are defined by connecting point in the force diagram. Diagonal web member 4-5 is made a zero force member.

Graphic Statics

How can we make the force in the top chord constant?



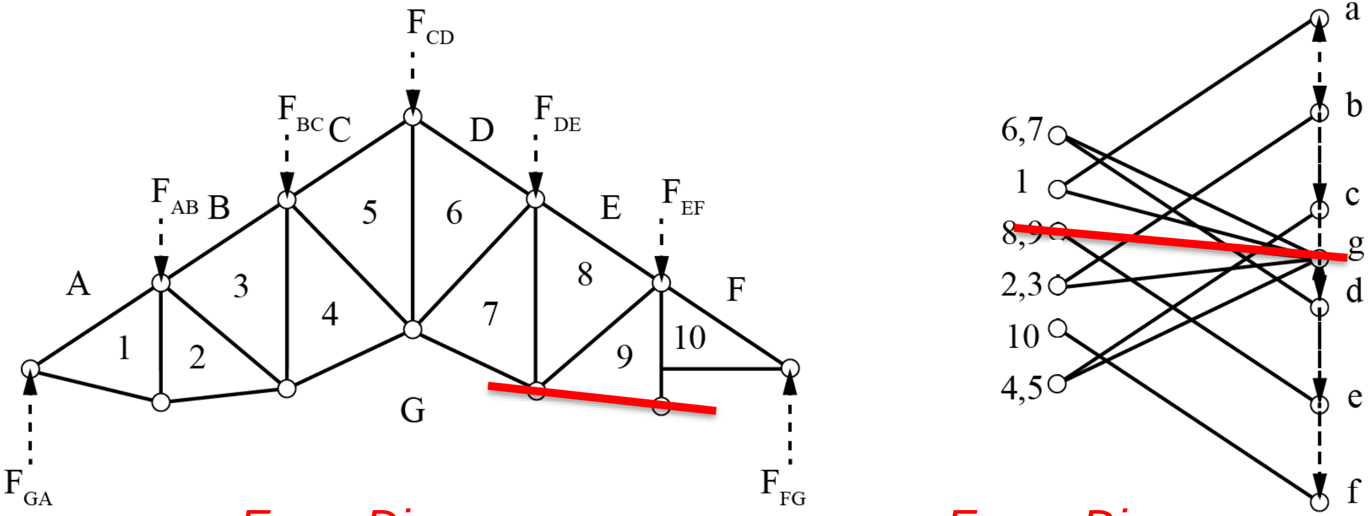
Form Diagram

Force Diagram

Directions of bottom chord members are defined by connecting point in the force diagram. Diagonal web member 6-7 is made a zero force member.

Graphic Statics

How can we make the force in the top chord constant?

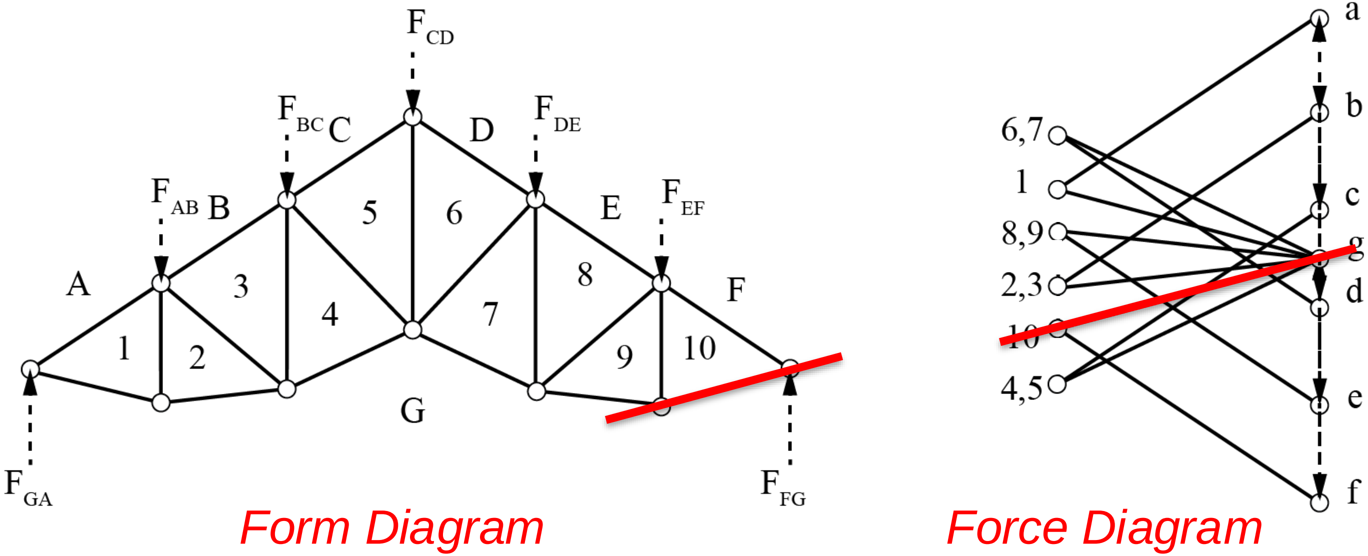


Form Diagram *Force Diagram*

Directions of bottom chord members are defined by connecting point in the force diagram. Diagonal web member 8-9 is made a zero force member.

Graphic Statics

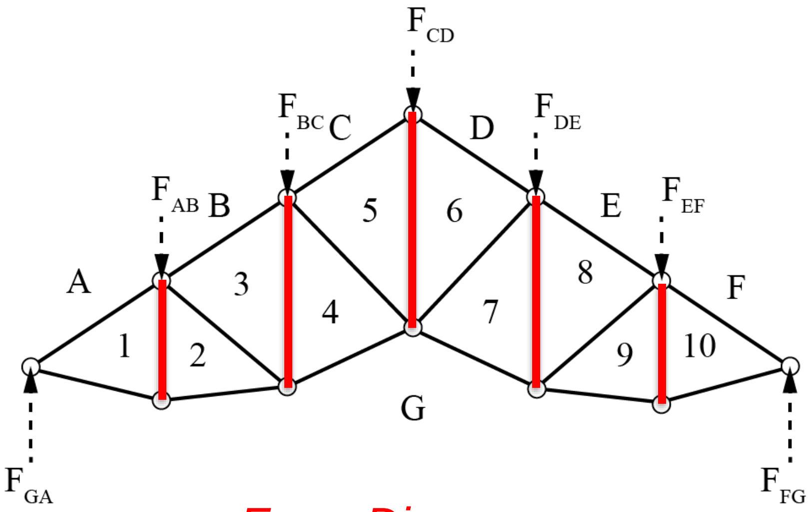
How can we make the force in the top chord constant?



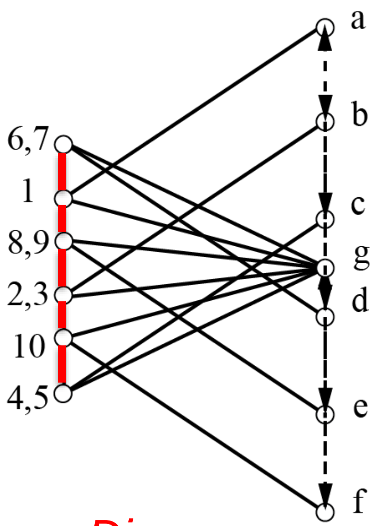
Directions of bottom chord members are defined by connecting point in the force diagram.

Graphic Statics

How can we make the force in the top chord constant?



Form Diagram

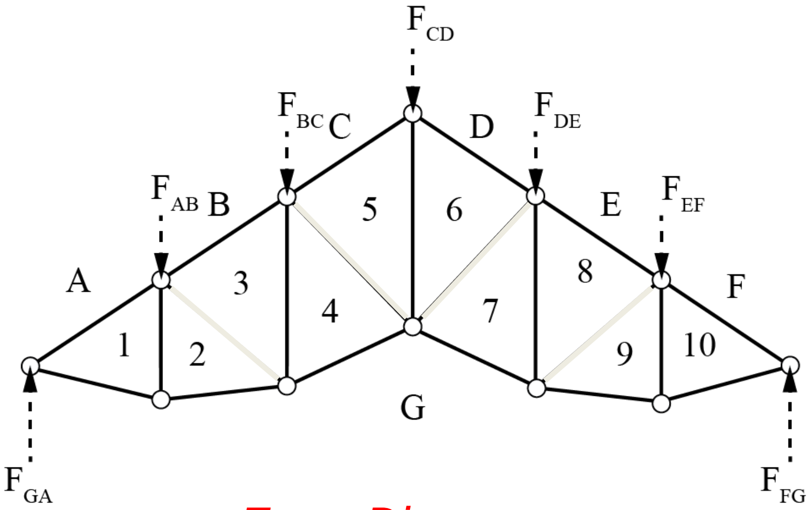


Force Diagram

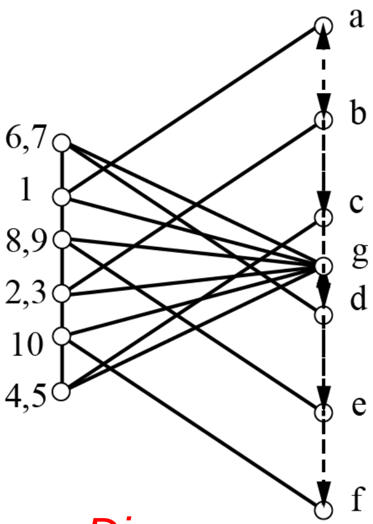
Vertical web members remain vertical

Graphic Statics

How can we make the force in the top chord constant?



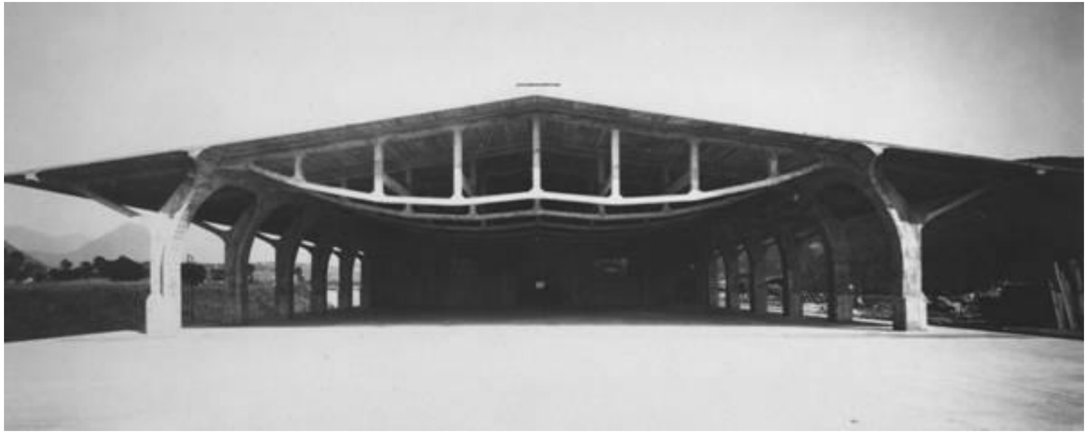
Form Diagram

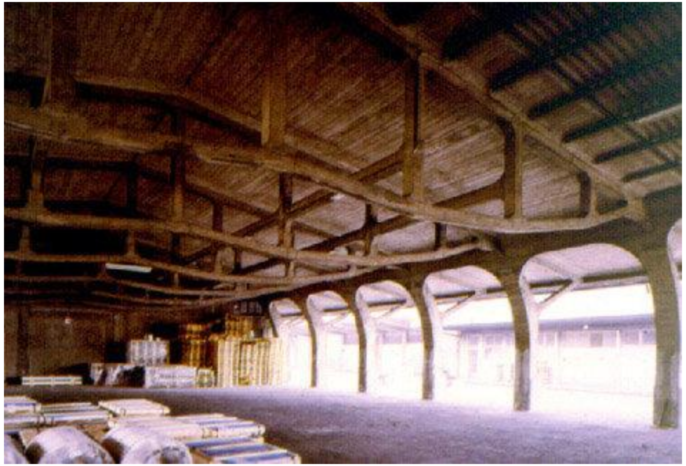


Force Diagram

Diagonal web members are zero-force.

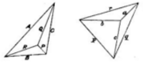
Constant-force gable truss



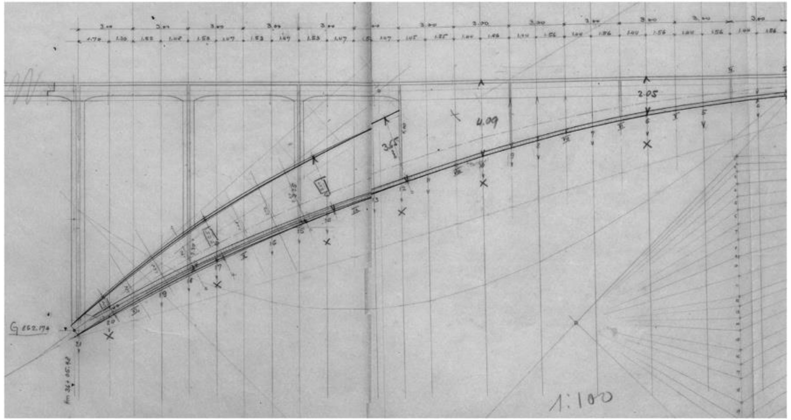


Magazzini Generali Warehouse
Robert Maillart, 1924

SHAPE

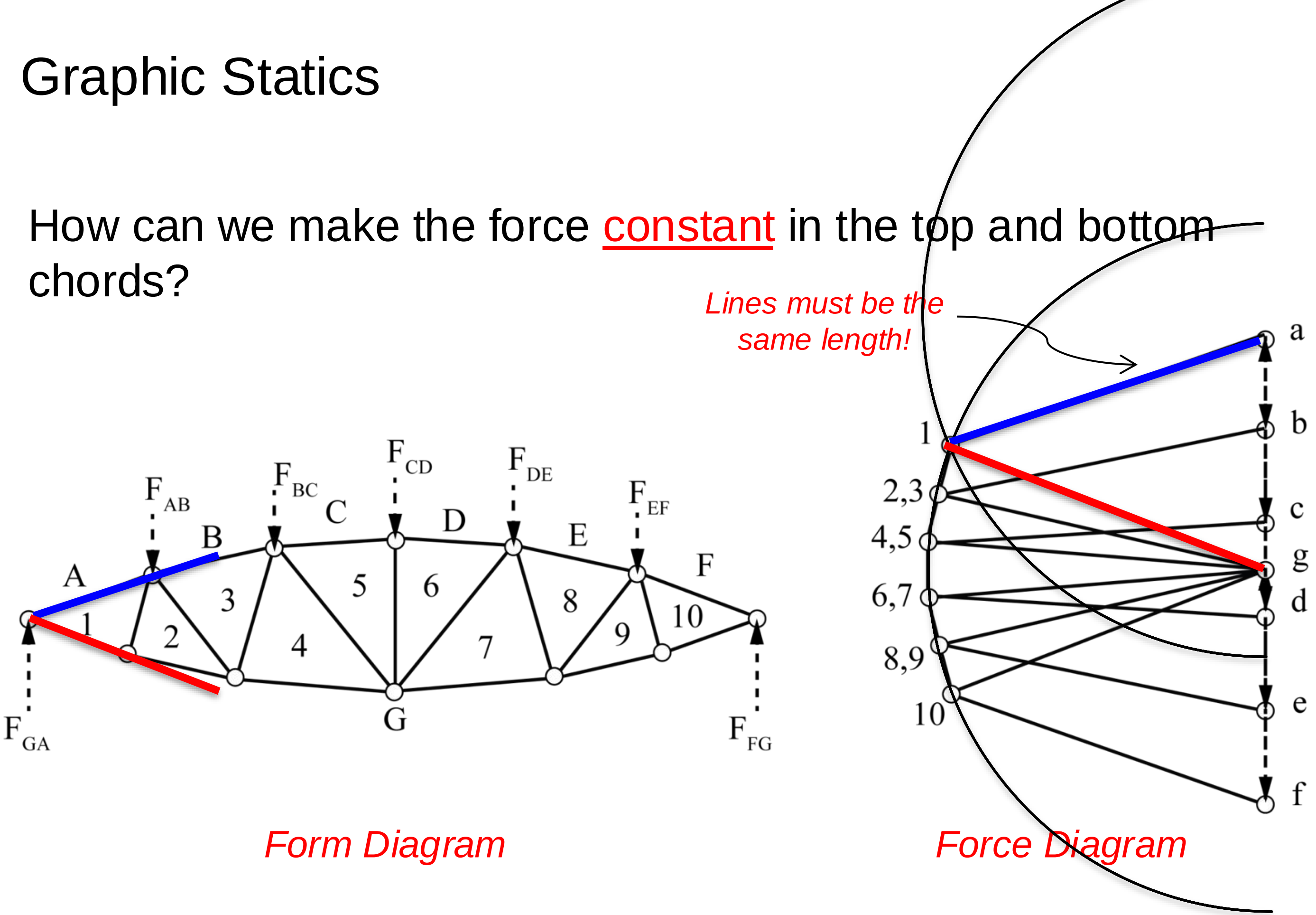


Graphics Statics



Graphic Statics

How can we make the force constant in the top and bottom chords?

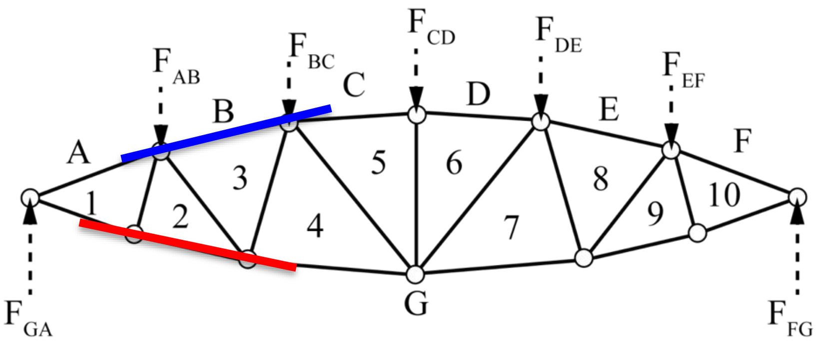


Form Diagram *Force Diagram*

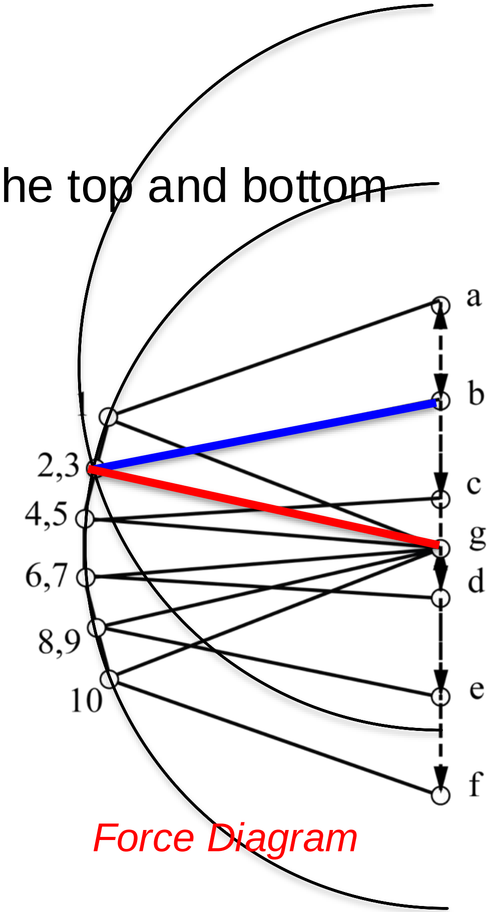
Modify the force diagram and work backwards!

Graphic Statics

How can we make the force constant in the top and bottom chords?



Form Diagram

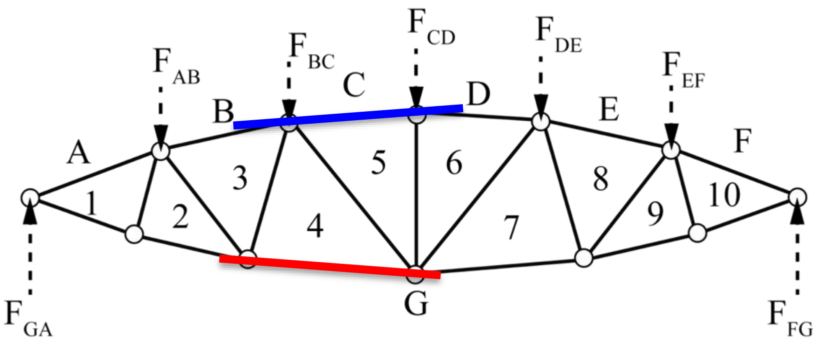


Force Diagram

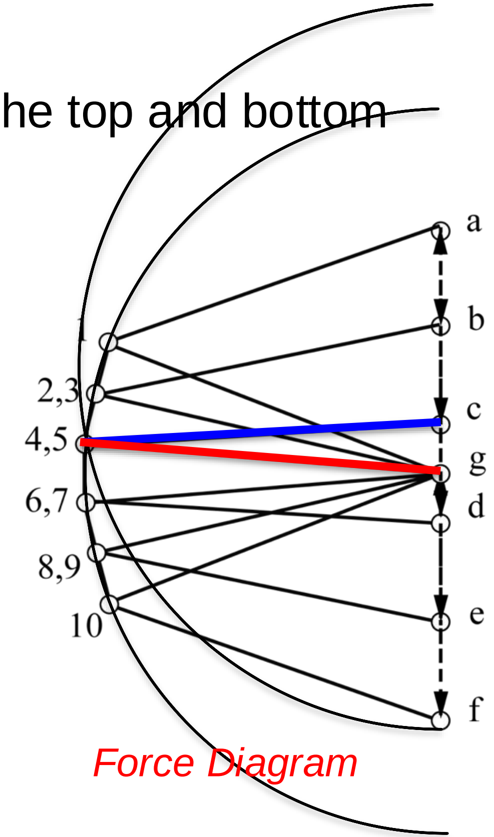
Modify the force diagram and work backwards!

Graphic Statics

How can we make the force constant in the top and bottom chords?



Form Diagram



Force Diagram

Modify the force diagram and work backwards!

FROM GRAPHIC STATICS TO AIRY STRESS FUNCTION 1864 1870

James Clerk Maxwell
1831 - 1879

Key Concept from Maxwell

Another such point can be determined by going round the remaining sides of the polygon; and these two points, together with the intersections of the lines A E, must all be in one straight line, namely, the intersection of the faces A P and E T.

Hence the conditions of the possibility of reciprocity in plane figures are the same as those of each figure being the perspective projection of a plane-sided polyhedron. When the number of points is in every part of the figure equal to or less than the number of polygons, this condition is fulfilled of itself. When the number of points exceeds the number of polygons, there will be an impossible case, unless certain conditions are fulfilled so that certain sets of intersections lie in straight lines.

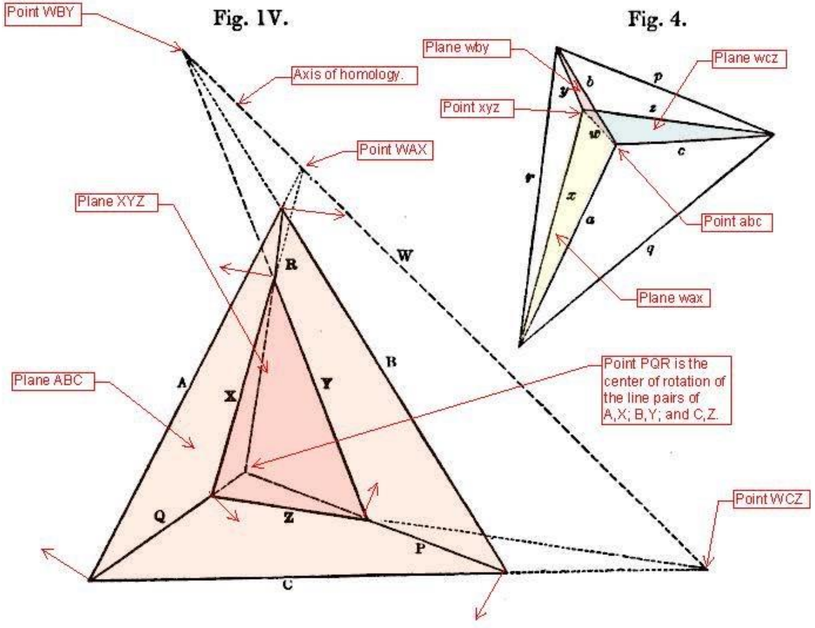
Application to Statics.

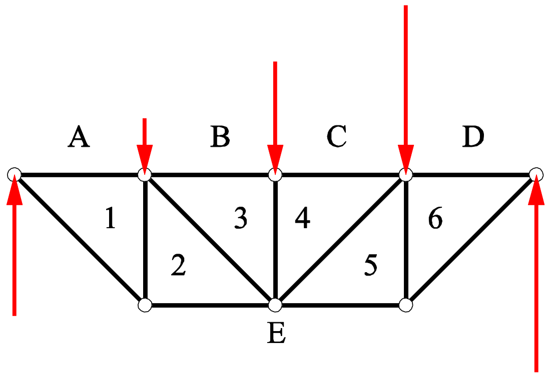
The doctrine of reciprocal figures may be treated in a purely geometrical manner, but it may be much more clearly understood by considering it as a method of calculating the forces among a system of points in equilibrium; for,

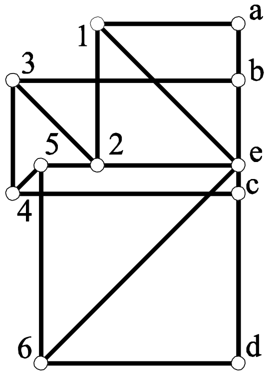
If forces represented in magnitude by the lines of a figure be made to act between the extremities of the corresponding lines of the reciprocal figure, then the points of the reciprocal figure will all be in equilibrium under the action of these forces.

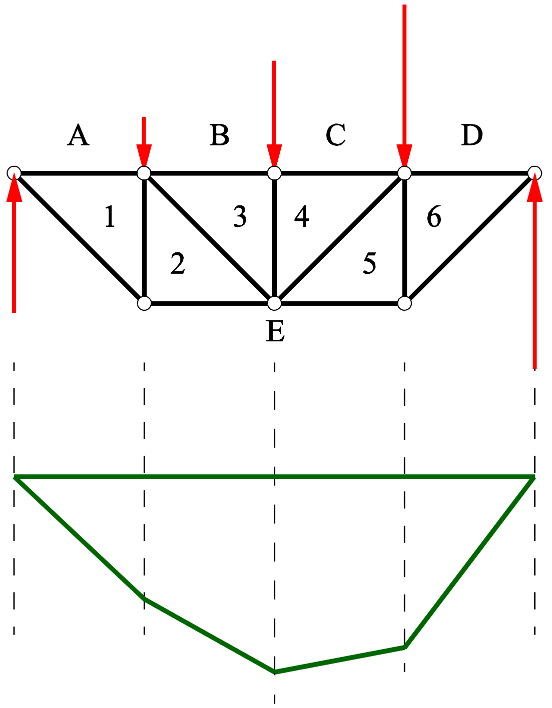
Mechanism W

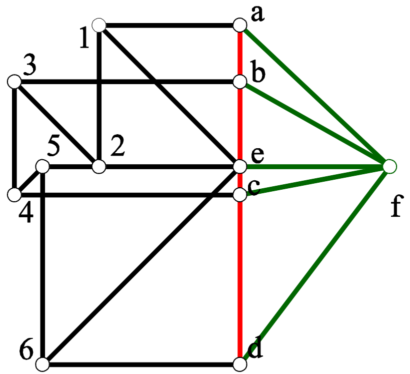
In figures 4 and IV. the condition that the number of polygons is equal to the number of points is not fulfilled. In fig. 4 there




A
B
C
D
1
3
4
6
2
5
E


1
a
3
b
5
2
e
c
4
6
d


A
B
C
D
1
2
3
4
5
6
E


a
b
e
c
d
f
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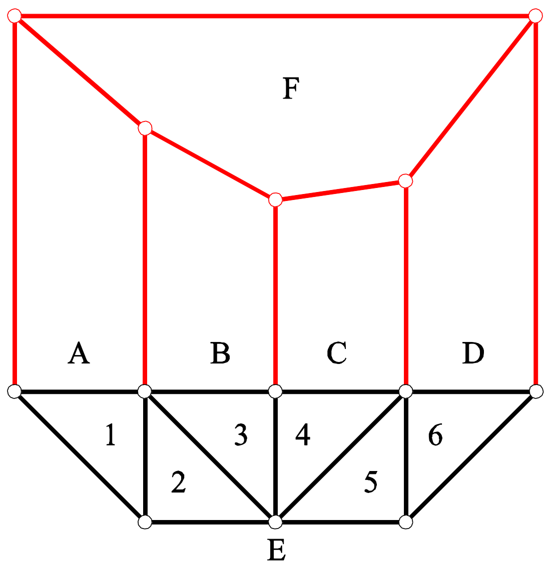
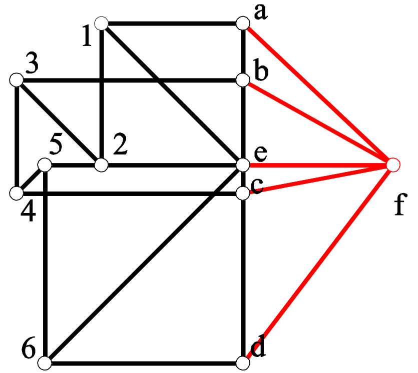
6


F
A
B
C
D
1
2
3
4
5
6
E


a
1
3
b
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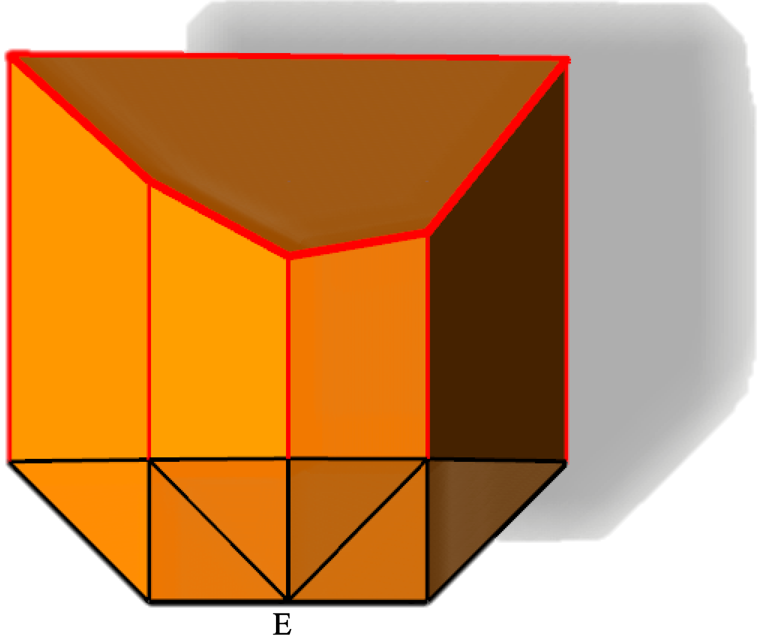
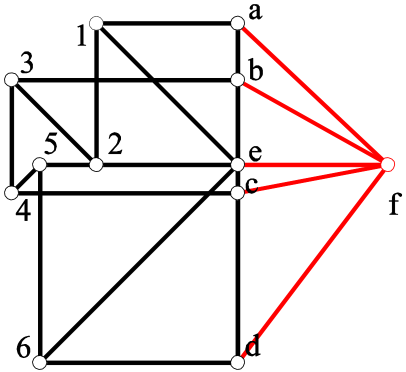
c
4
f
6
d


E


a
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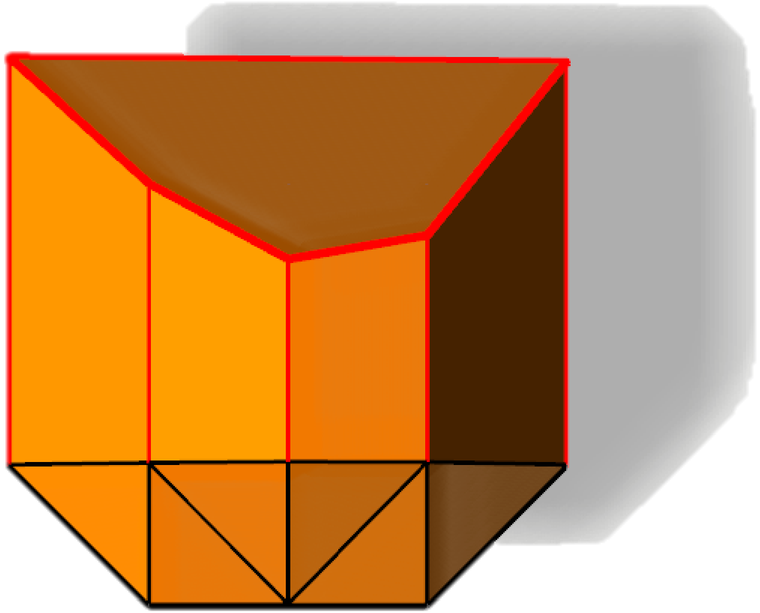
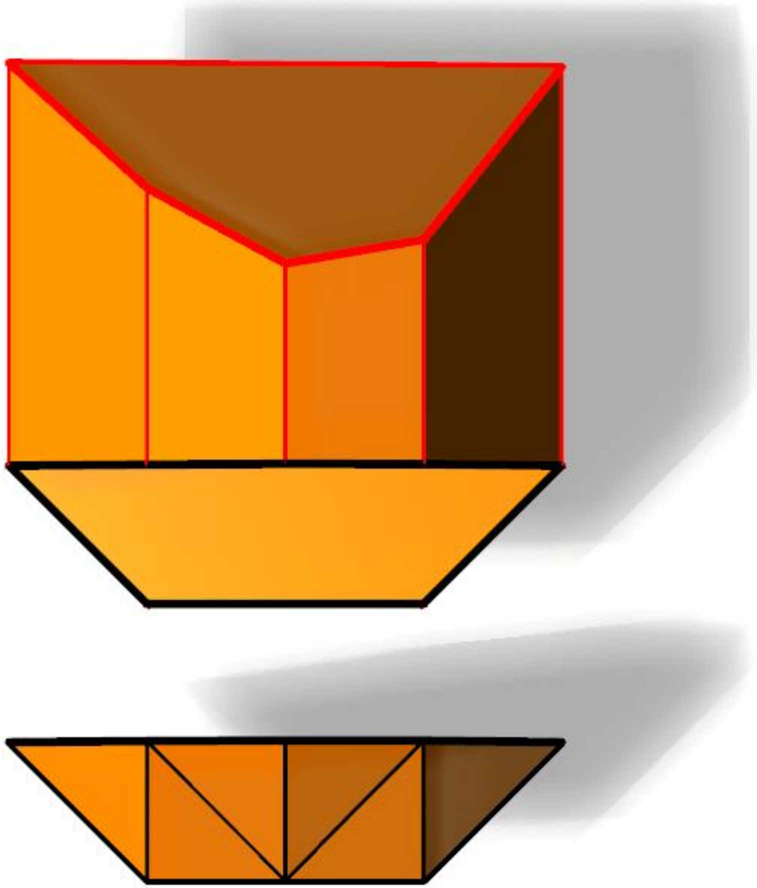
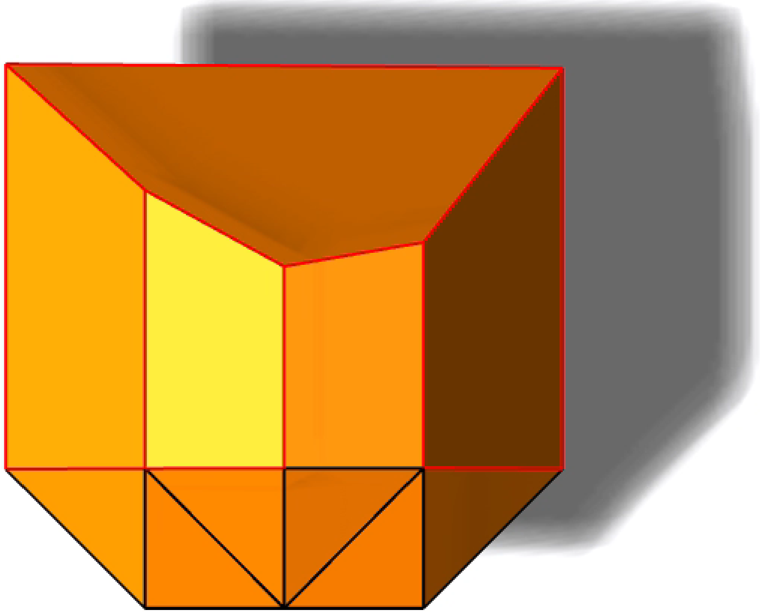
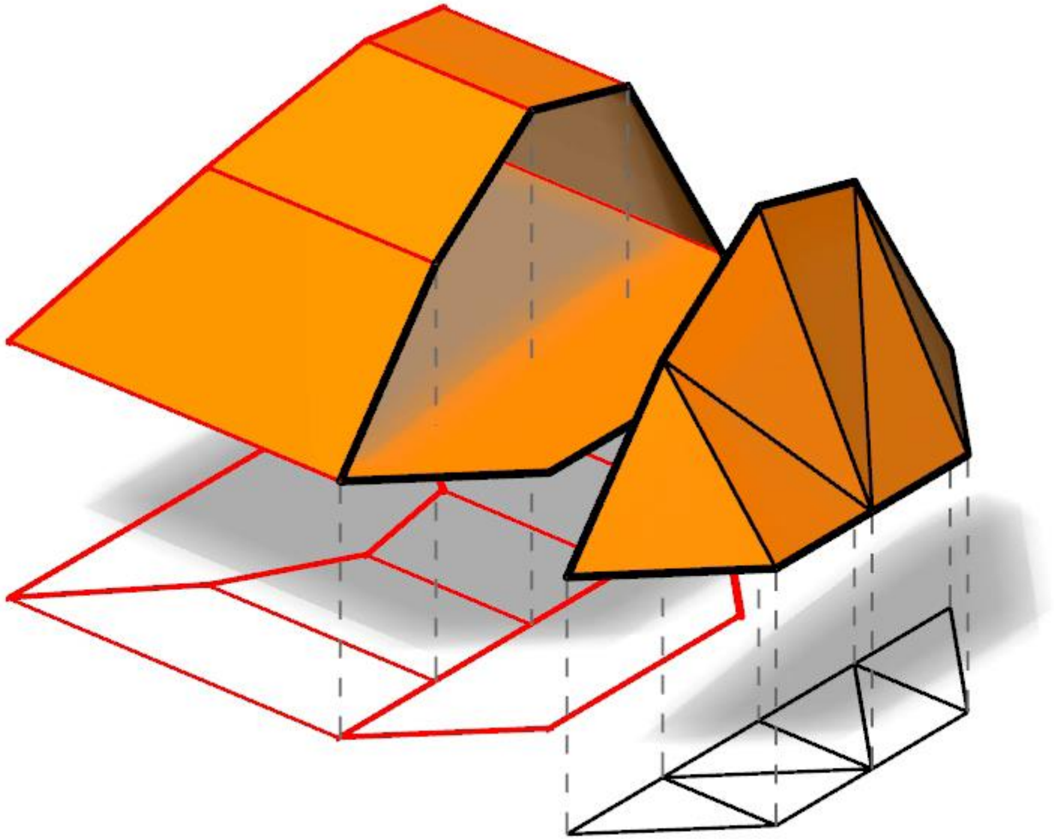
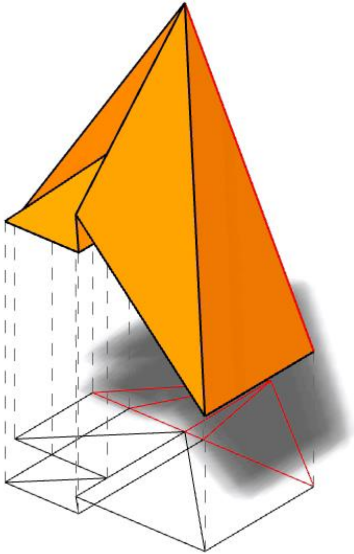
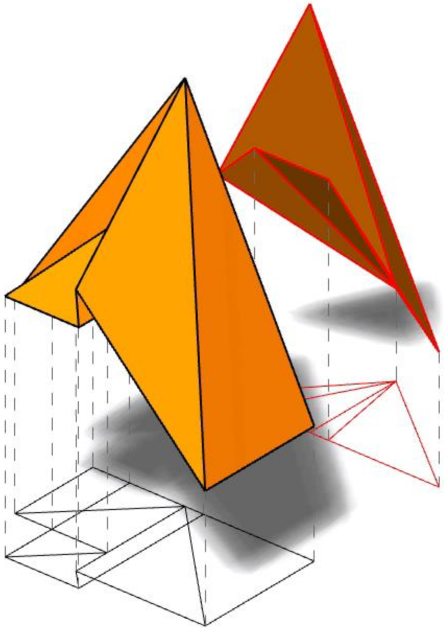












DESIGN OF GRIDSHELLS

BRITISH MUSEUM GREAT COURT ROOF
FOSTER + PARTNERS, BURO HAPPOLD,
WAAGNER BIRO, PROF CHRIS JK WILLIAMS

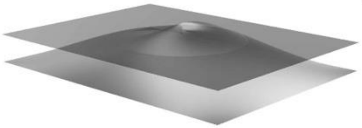


Figure 2. Level change function,

$$\frac{\left(1-\frac{x}{b}\right)\left(1+\frac{x}{b}\right)\left(1-\frac{y}{c}\right)\left(1+\frac{y}{d}\right)}{\left(1-\frac{ax}{rb}\right)\left(1+\frac{ax}{rb}\right)\left(1-\frac{ay}{rc}\right)\left(1+\frac{ay}{rd}\right)}$$

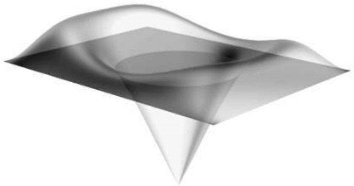


Figure 3. Function with finite curvature at corners

$$\left(\frac{r}{a}-1\right)\left(1-\frac{x}{b}\right)\left(1+\frac{x}{b}\right)\left(1-\frac{y}{c}\right)\left(1+\frac{y}{d}\right)$$

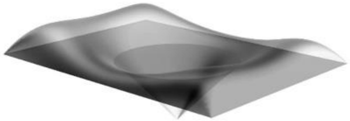


Figure 4. Function with conical corners

$$\frac{1-\frac{a}{r}}{\frac{\sqrt{(b-x)^2+(c-y)^2}}{(b-x)(c-y)}+\frac{\sqrt{(b-x)^2+(d+y)^2}}{(b-x)(d+y)}+\frac{\sqrt{(b+x)^2+(c-y)^2}}{(b+x)(c-y)}+\frac{\sqrt{(b+x)^2+(d+y)^2}}{(b+x)(d+y)}}$$

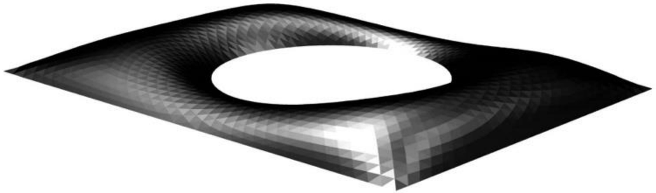


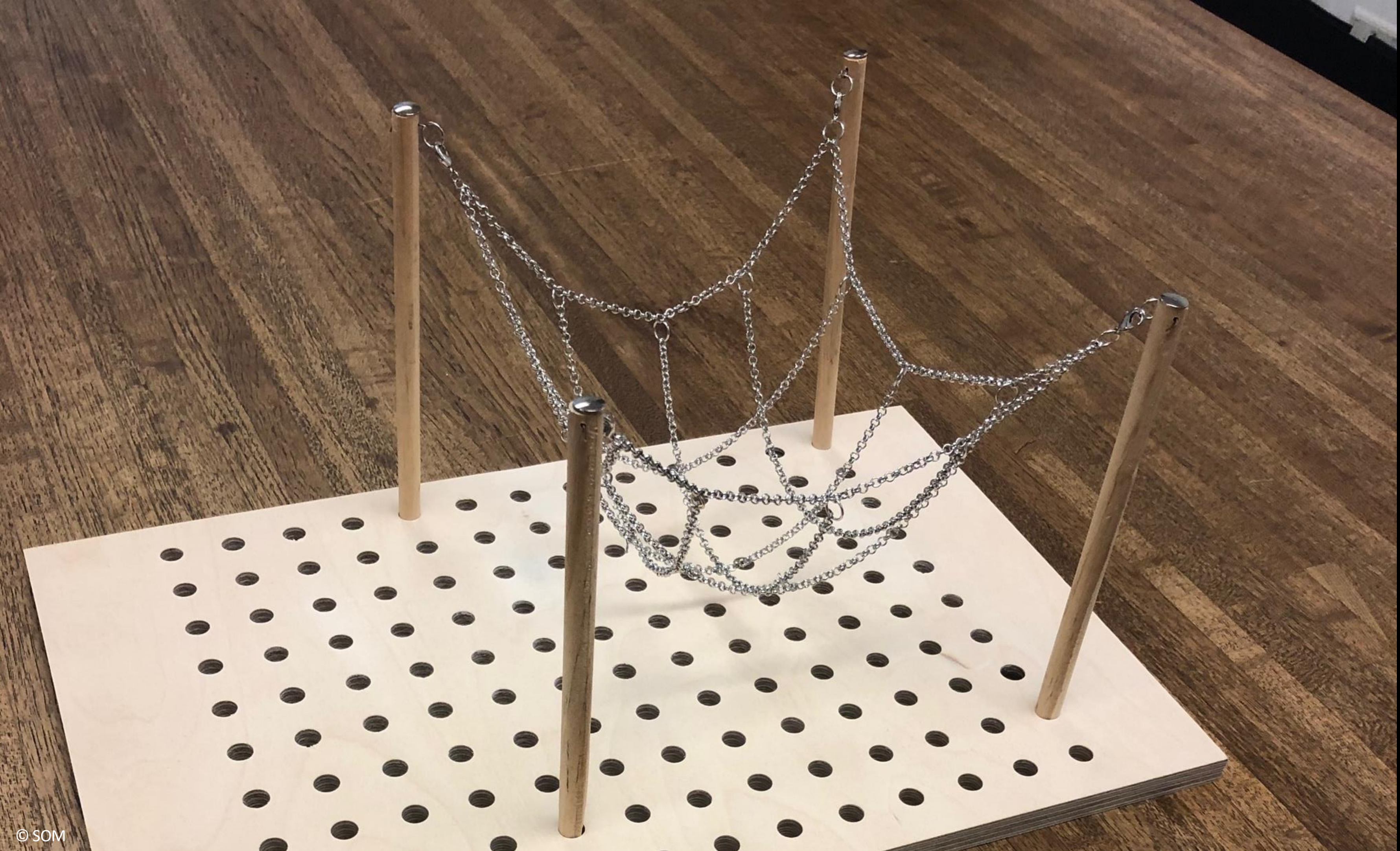
Figure 5. Final surface

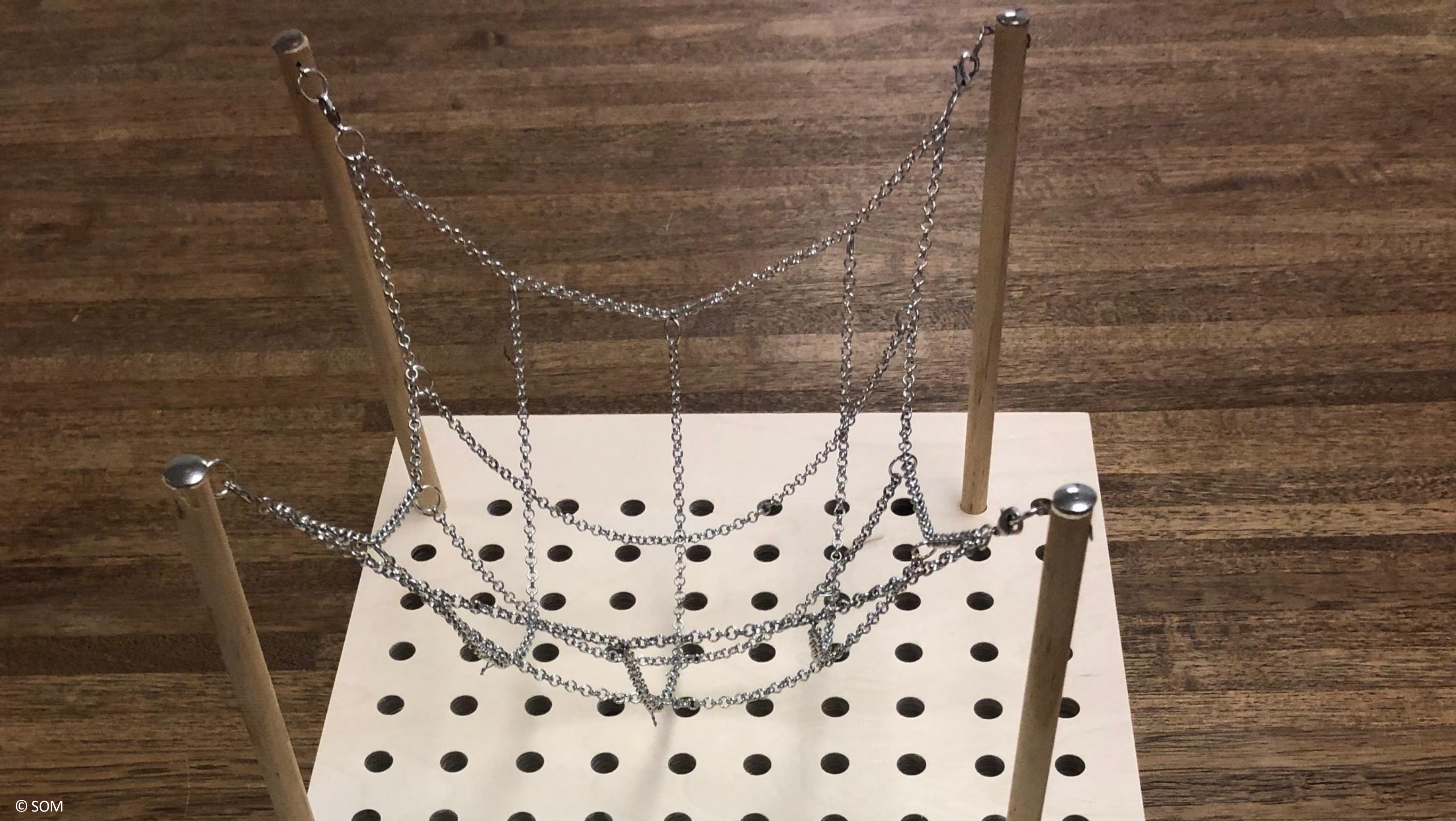
BASIC QUESTION:

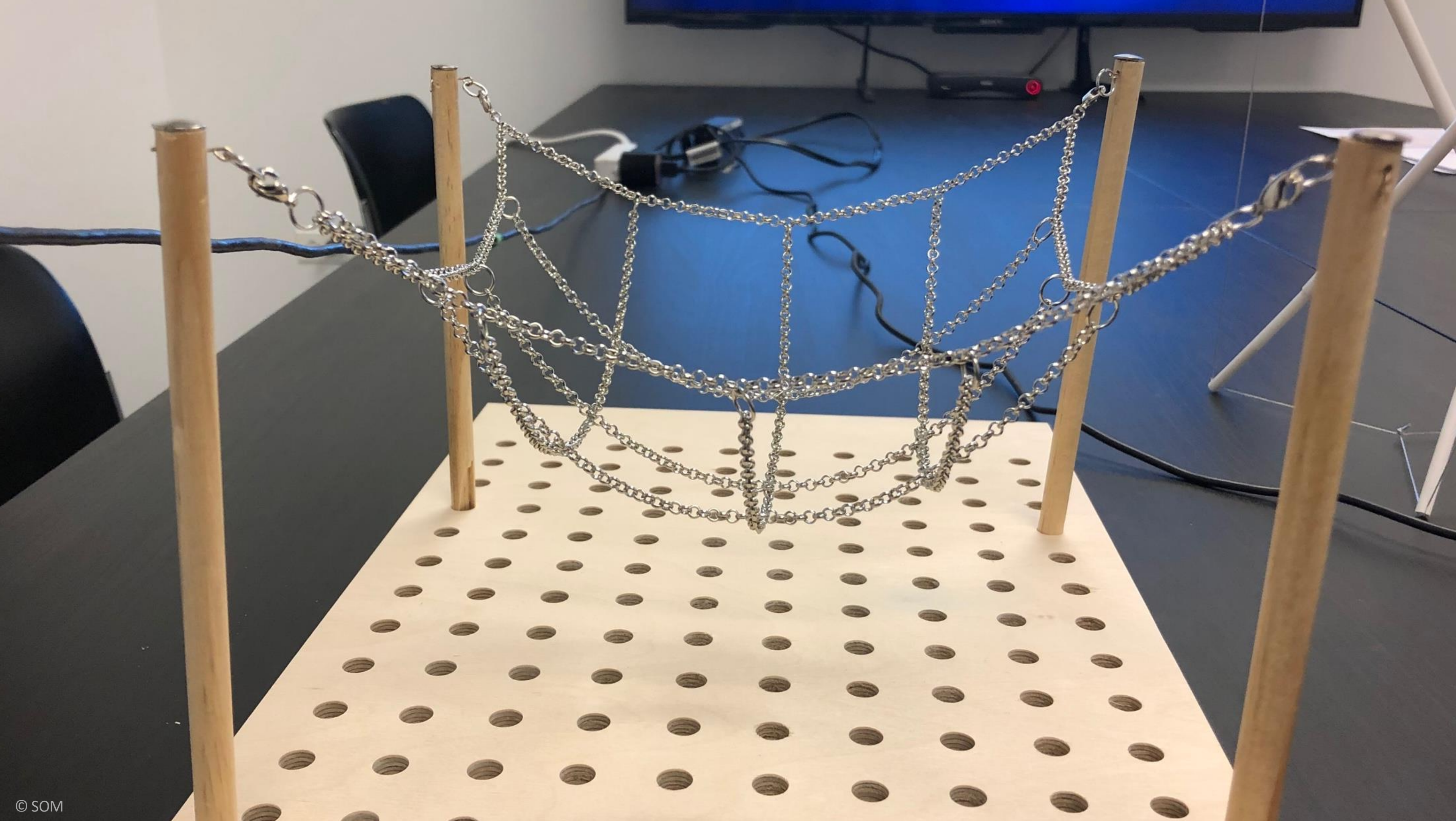
CAN WE GET BEYOND ONE OF OUR GREATEST LIMITATIONS?

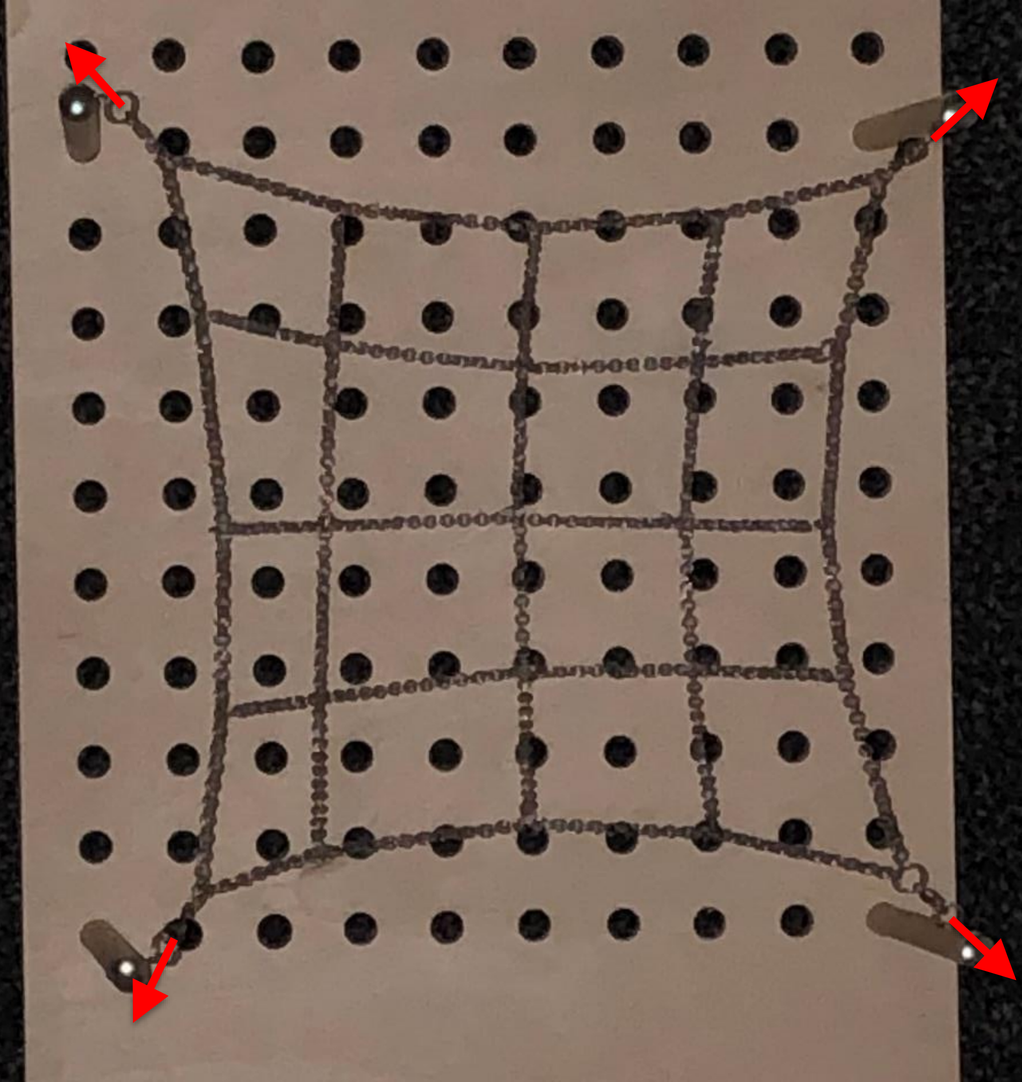
THERE IS ONLY ONE
CHRIS WILLIAMS

2D PROJECTION OF A 3D EQUILIBRIUM

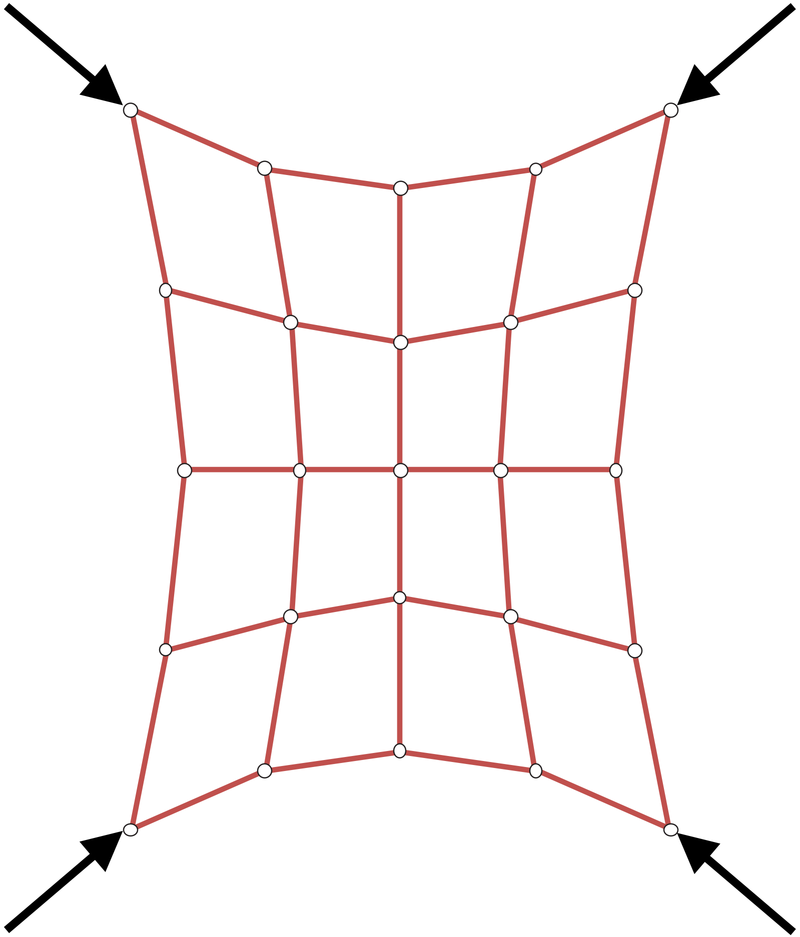




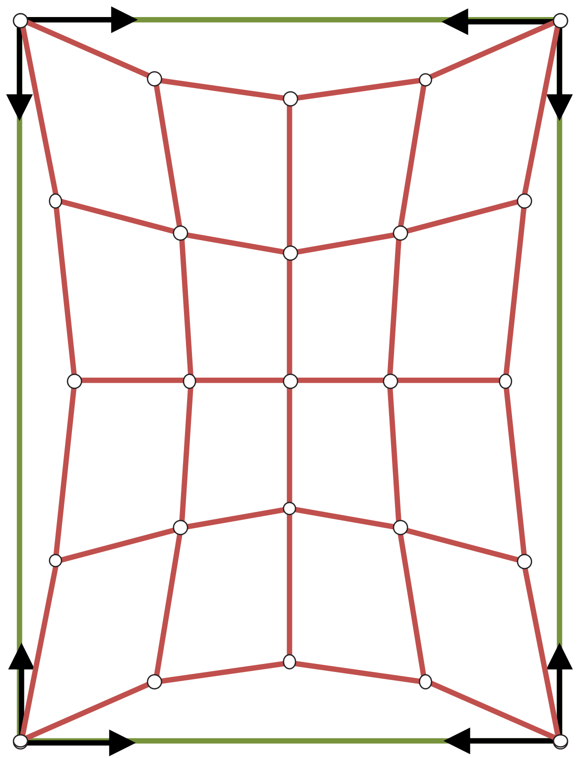




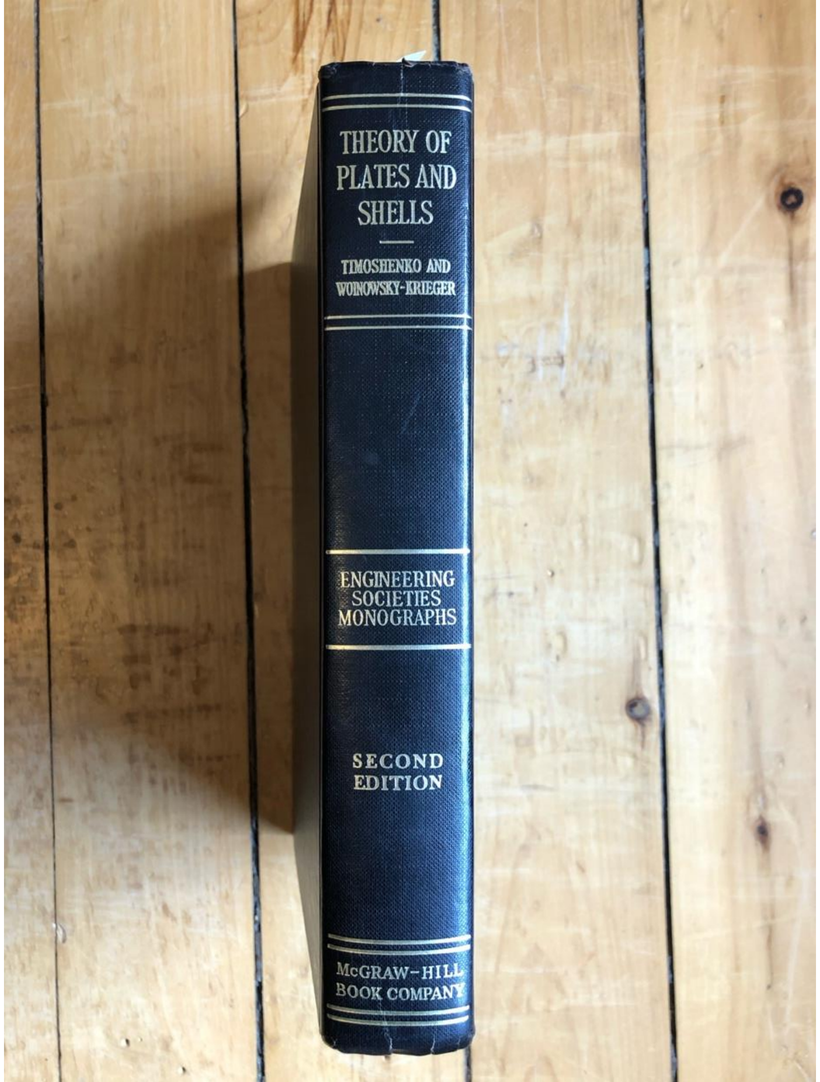
Plan layout spanning axially with abutments



Plan layout spanning axially with prestressed ring

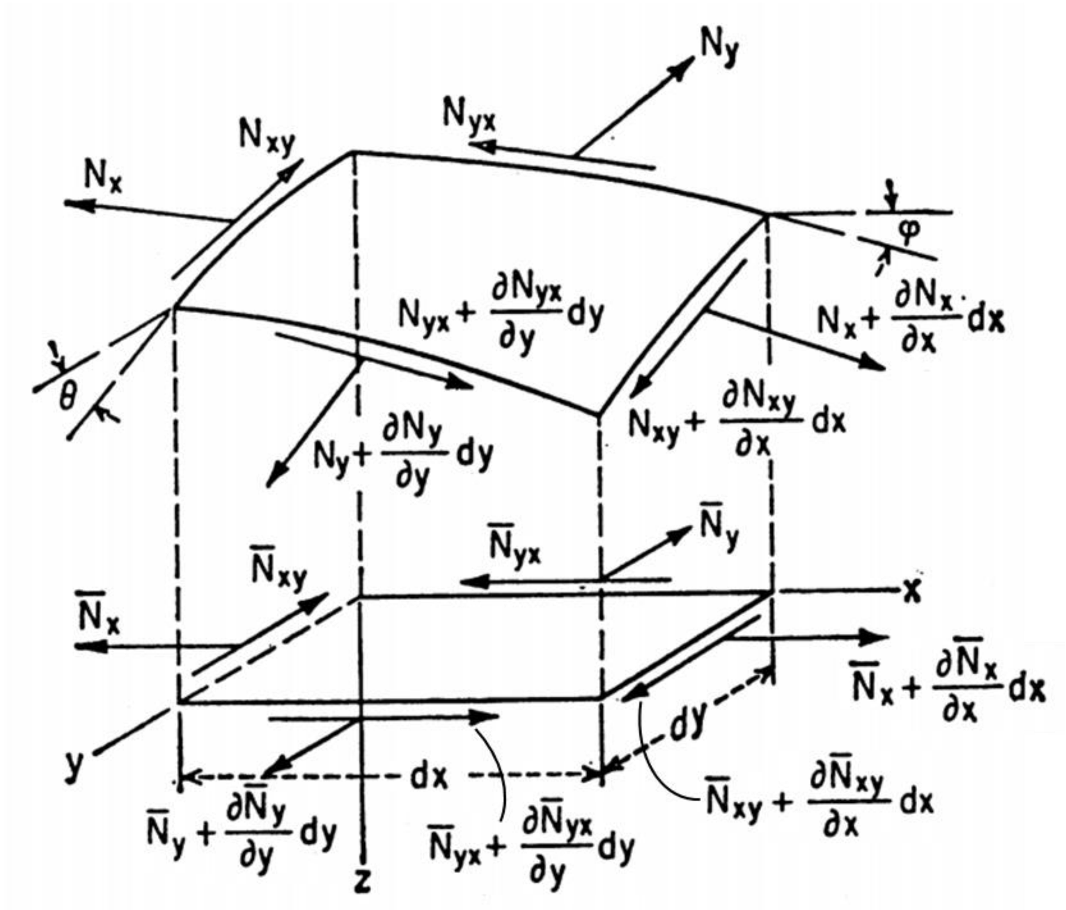


TIMOSHENKO'S AND PUCHER'S PLATES AND SHELLS

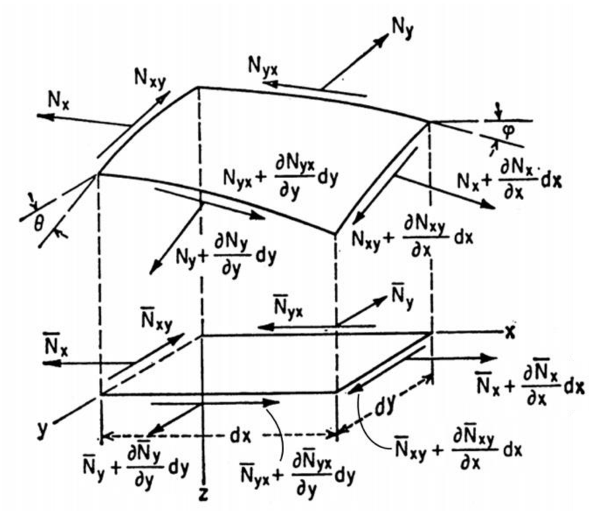


…although Pucher discovered the theory

Shell Equation



Shell Equation



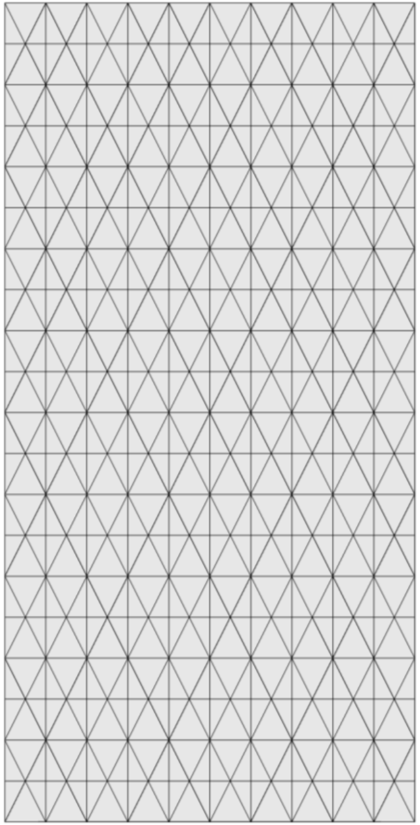
$$\frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 z}{\partial x^2} - 2\,\frac{\partial^2 \phi}{\partial x \partial y}\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 z}{\partial y^2} = F'$$

Timoshenko or Pucher shell equation

DESIGNING AN AIRY STRESS FUNCTION

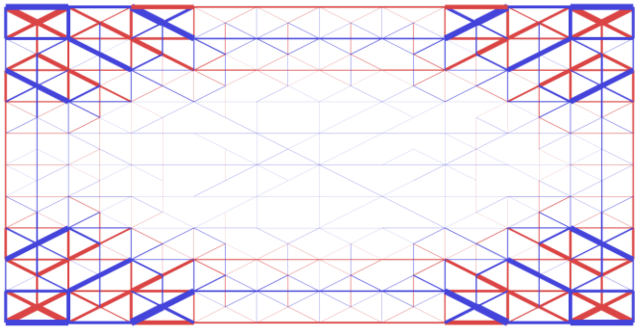
Simple Triangulated Gridshell

Form diagram

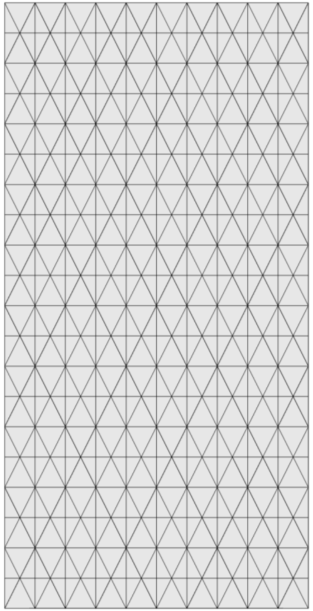


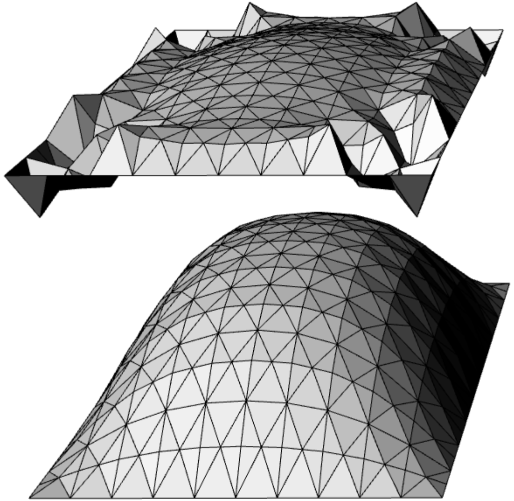
Note that it is triangulated so that it can thrust into the corners.

Simple Triangulated Gridshell



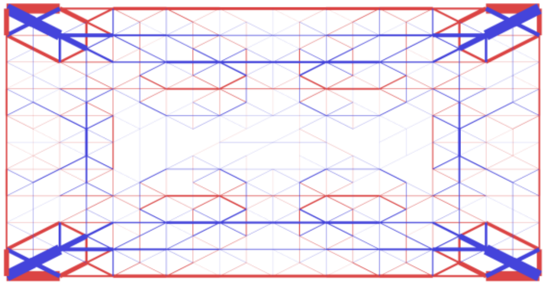
Form diagram



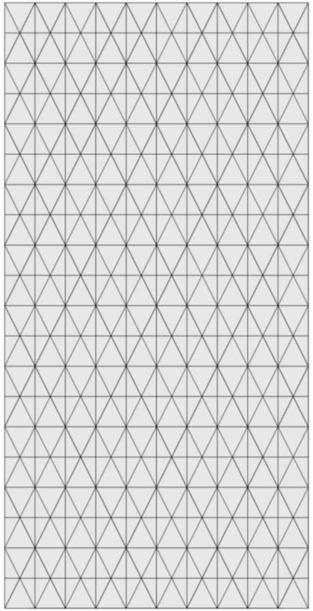


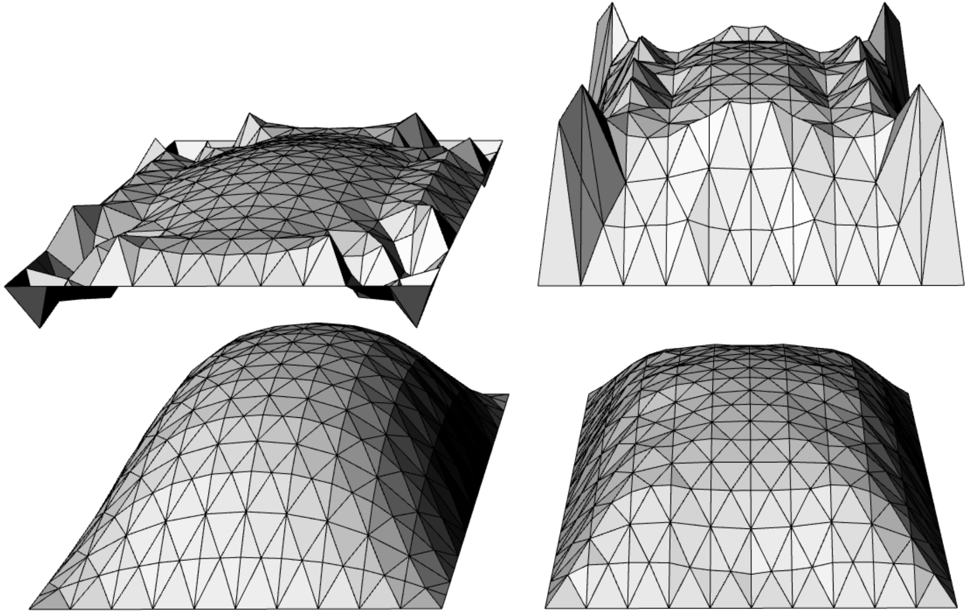
Negative Gaussian curvature at the corner causes tension.

Simple Triangulated Gridshell



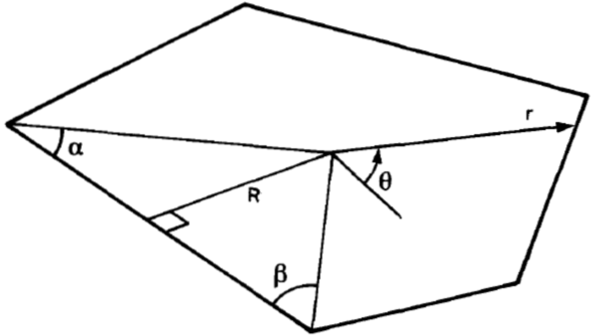
Form diagram



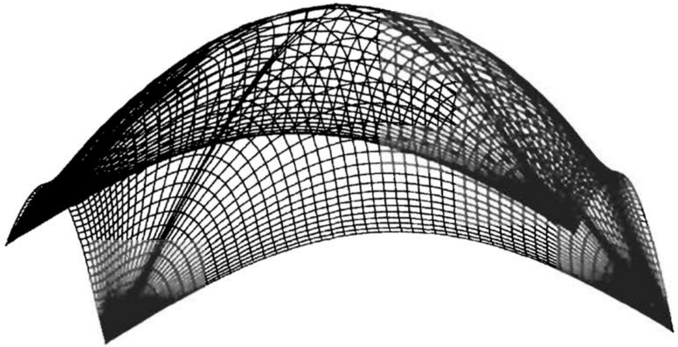


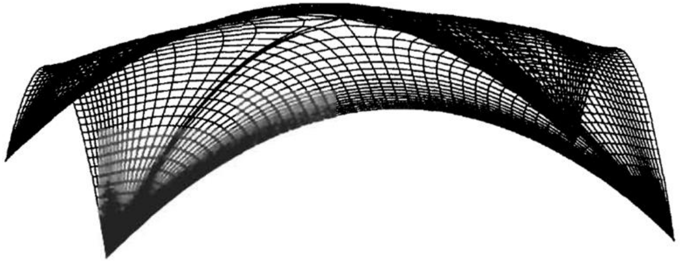
Triangulated Gridshells

One can obtain a good initial geometry using an analogy with the Biot-Savart law.

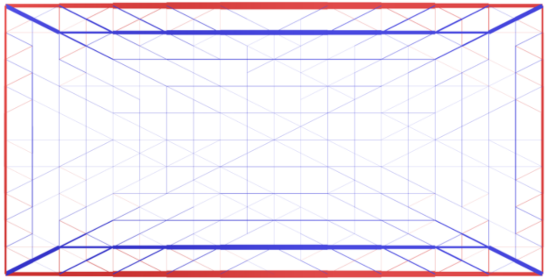


$$\phi = \frac{C}{\sum \frac{\cos\alpha + \cos\beta}{R}}$$

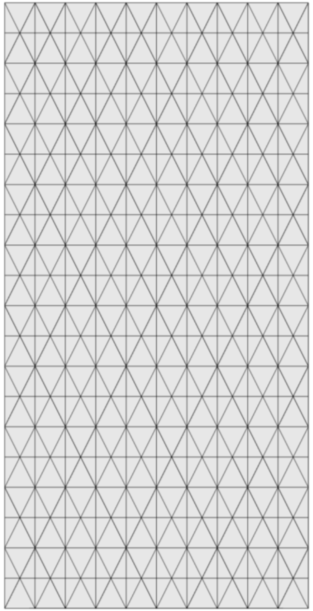

(a)

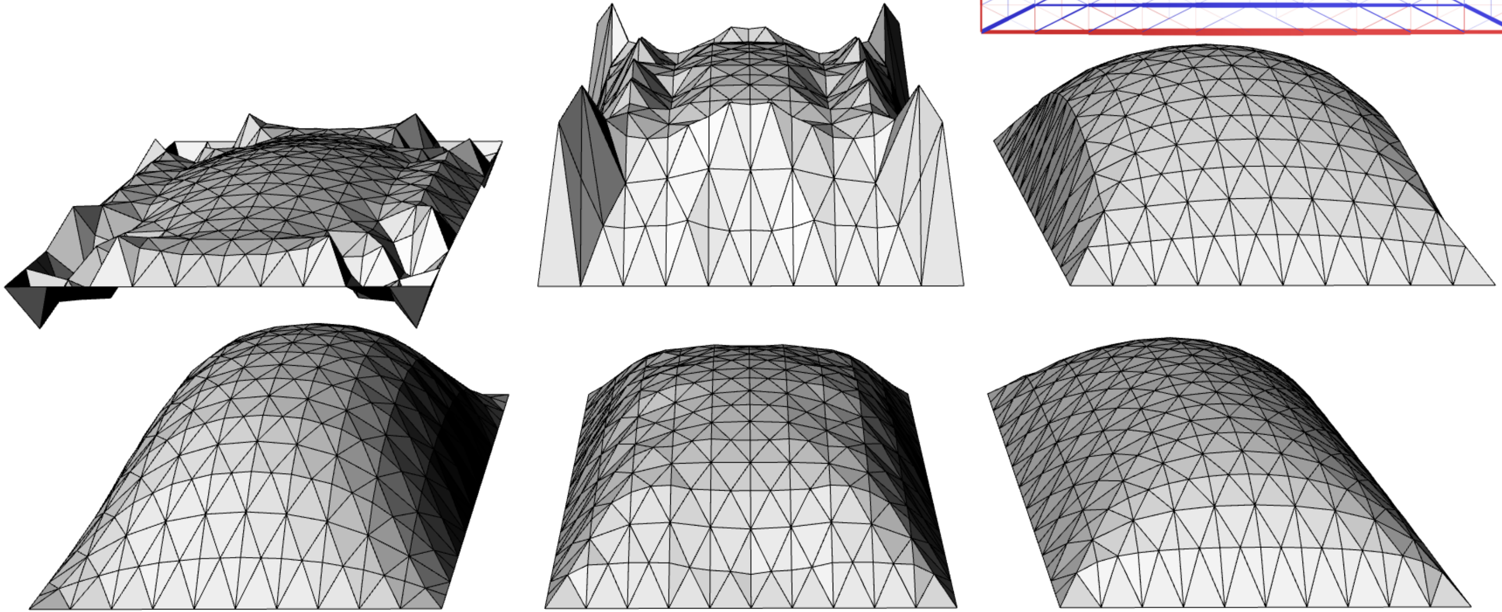

(b)

Simple Triangulated Gridshell



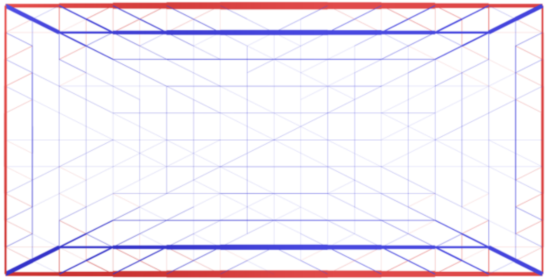
Form diagram



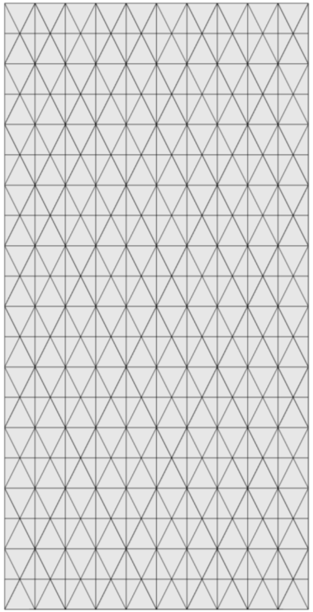


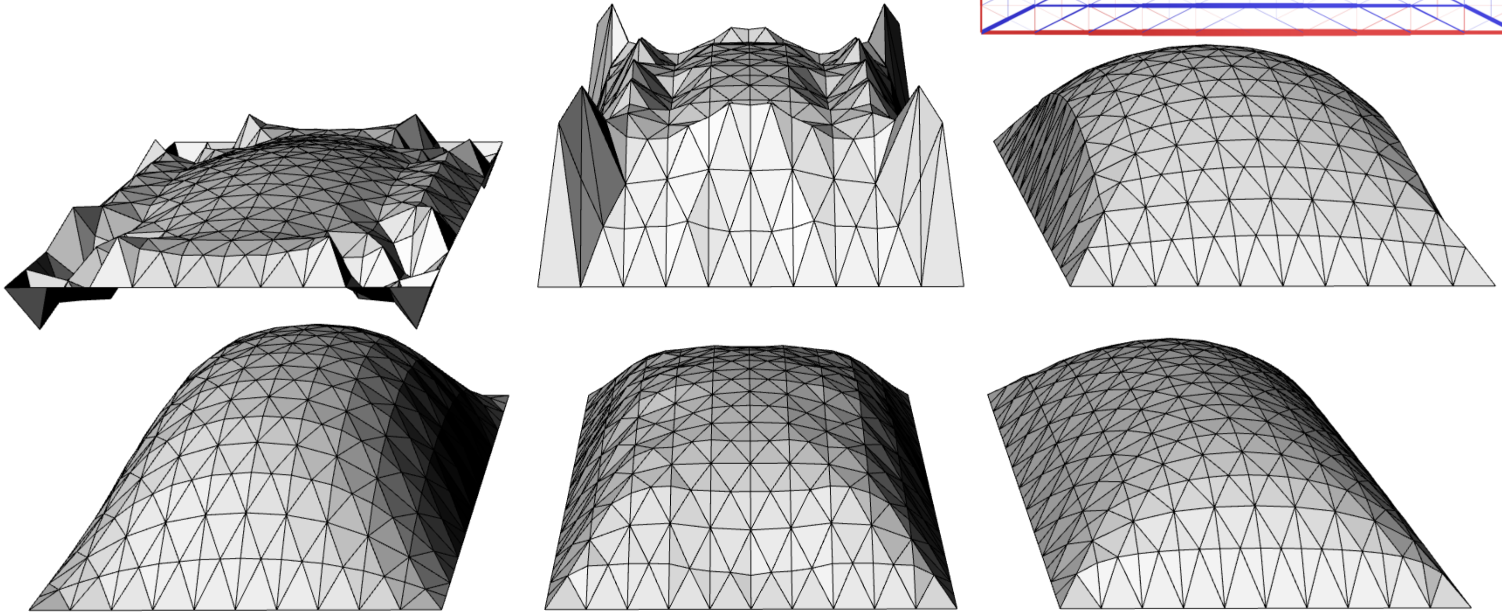
Good engineering properties – little tension and uniform forces.

Simple Triangulated Gridshell



Form diagram





Remember, the roof and Airy stress function can be swapped!

Structural Innovation: Combining Classic Theories with New Technologies

WILLIAM F. BAKER, LAUREN L. BEGHINI, ARKADIUSZ MAZUREK, JUAN CARRION and ALESSANDRO BEGHINI

Baker received AISC's T.R. Higgins Lectureship Award in 2013. Learn more about this prestigious award at www.aisc.org/higgins.

ABSTRACT

In the early stages of the design process, an engineer sets the geometry of the structure. The decisions made about the layout of the structure will determine the overall efficiency that can be achieved and the magnitude of the forces that must be accommodated. For example, the deflection of a truss can actually be decreased by removing material if a geometry is created that has a shorter total load path. This paper presents concepts and methodologies for creating and understanding efficient geometries. It starts with a review of the 19th- and 20th-century load path theories of Rankine, Maxwell, Cremona and Michell. It then combines their insights with current topology optimization and shape-finding tools as a means of exploring how engineers can create structural geometries that improve the behavior and reduce the tonnage of their designs. Several examples of classical theoretical solutions are explored along with their application to new designs.

Keywords: structural geometry, structural efficiency, structural analysis.

INTRODUCTION

The success of any project depends on starting with a good concept. For the design of structural steel trusses and other structures, the geometrical arrangement of the members is often the most important consideration in producing an efficient and well-behaved design. Although efficiency has always been a chief design consideration, its importance has increased lately as designers seek to minimize the carbon footprint in the construction of new structures. Where can the designer seek guidance in creating layouts that achieve the goals of efficiency and good behavior? A good place to begin is at the start of modern structural engineering.

The mid-19th century was a key period in the advancement of the understanding of structural behavior. The theory of elasticity had already been highly advanced through the development of elastic "aether" theories, and many mathematicians, scientists and natural philosophers were extending their studies into structural mechanics, as well as optics, electricity and magnetism. Their interest in structures was undoubtedly further influenced by the advent of the railroad.

The emergence of railroads led to technological challenges and advancements. The railroads needed bridges and, as a response, the first metal truss bridge was built in the United States in 1840 and in the United Kingdom in 1845 (Timoshenko, 1953). The great thinkers of the time began focusing their thoughts upon the practical issues of trusses and bridges and, in doing so, pushed the limits of structural engineering. One such example is the British Astronomer Royal, George Biddell Airy, who not only studied the stars, but also developed his famous Airy stress function in response to Stephenson's Britannia Bridge (Airy, 1863).

This paper reviews some important works by Rankine, Maxwell, Cremona and Michell that still have great relevance to modern design. Today's structural engineer can combine the ideas of these great innovators with modern topology optimization tools to develop structural concepts for steel trusses and other structures. By combining these concepts with practical considerations of constructability and cost, the structural engineer can develop responsible designs that can minimize the carbon footprint in the construction of new structures and help reduce the consumption of our natural resources.

Please note the theories and findings included in this paper are based on equilibrium and compatibility and, when calculating volume, strength or deflection, constitutive relationships assuming linearly elastic material. The analysis and exploration of the effects of geometric and material nonlinearites on optimal topology layouts is under investigation by a number of researchers.

William F. Baker, P.E., S.E., Partner, Skidmore, Owings & Merrill LLP, Chicago, IL (corresponding). Email: william.baker@som.com

Lauren L. Beghini, Ph.D., Structural Engineer, Skidmore, Owings & Merrill LLP, Chicago, IL. Email: lauren.beghini@gmail.com

Arkadiusz Mazurek, Ph.D., P.E., S.E., Associate, Skidmore, Owings & Merrill LLP, Chicago, IL. Email: arkadiusz.mazurek@som.com

Juan Carrion, Ph.D., P.E., Structural Consultant, Skidmore, Owings & Merrill LLP, Chicago, IL. Email: juan.carrion@som.com

Alessandro Beghini, Ph.D., P.E., S.E., Associate, Skidmore, Owings & Merrill LLP, San Francisco, CA. Email: alessandro.beghini@som.com

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